Self-fulfilling Liquidity Dry-ups

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Abstract

In this paper, I consider an economy in which investors face a return-liquidity trade-off: long-term investment is on average more productive but liquidation on the secondary market is endogenously costly because of adverse selection. I find that: (1) when agents expect a liquidity dry-up, they optimally choose to self-insure through the hoarding of non-productive but liquid assets; (2) such a response reduces ex-post market participation which worsen adverse selection and causes the anticipated dry-up to happen; (3) liquidity dry-ups are Pareto inefficient equilibria; (4) the Government can rule them out. A public liquidity insurance scheme implements the second-best because the prospect of a bail-out improves the return-liquidity trade-off. This prevents wasteful self-insurance, boosts long-term investment and market participation which, in turn, feeds liquidity back.

1 Introduction

During financial crises, the scope of public intervention is usually limited by moral hazard concerns. This is indeed well known that public intervention might induce investors to take on too much risk in the future[1] and sow then the seeds of the subsequent crisis. However, public intervention might also improve expectations about market liquidity and avoid socially costly liquidity dry-ups. This is what I show in this paper.

From an economic point of view, the liquidity of an asset is the ease by which it can be transformed into consumption good. In line with this, and as trading is never unilateral, market liquidity might be defined as the ease for a seller to find a counterpart that wants to trade at "fair" prices. A fair price might be defined as the expected (stochastically) discounted value of future payoffs[2]. Under common knowledge about the joint distribution of payoffs and discount factors, the market price is generally fair. Assets are thus liquid in such models. Clearly, exogenous frictions might impact liquidity; this is pretty well documented in the literature (see Amihud, Mendelson and Pedersen (2006) for a good review). Still, the case of endogenously illiquid assets has received much less attention.

Endogenous liquidity in asset markets is however introduced by Eisfeldt (2004)[3]. In her model, claims to long-run illiquid projects are traded at a discount with respect to the fair price due to adverse selection. In fact, it is endogenously costly to sell a claim to a good-quality ongoing project because the average quality of the

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[1]See for instance Diamond (1984) and Holmström & Tirole (1997).

[2]Cochrane (2001) shows that any asset pricing model might be reduced to: $P_t = E_t[m_{t+1}X_{t+1}]$ where X_{t+1} is the asset next period value, m_{t+1} a stochastic discount factor, and $E_t[.]$ mathematical expectation, conditional upon information available in period t.

[3]See also Brunnermeier and Pedersen (2005, 2009)

claims sold will be lower[4]. Still, there can be reasons (e.g. consumption smoothing in the case of a negative income shock) to sell high quality claims, even at a discount. The market clearing price is thus determined by the average seller's motive for trading. Translated into the definitions above: the higher the discount one concedes to sell a good asset, the lower its liquidity. Similarly, the higher the proportion of agents trading for consumption smoothing purpose (instead of private information), the lower the adverse selection and the higher the liquidity of the market.

In this paper, I consider an economy in which investors face a return-liquidity trade-off: long-term investment is on average more productive but liquidation on the secondary market is endogenously costly because of adverse selection. When agents expect the market to be illiquidity, they optimally choose to self-insure through the hoarding of non-productive but liquid assets. Such a response reduces ex-post market participation which worsen adverse selection and dries market liquidity up. I derive the general conditions under which such an outcome is Pareto inefficient and I show how the Government can rule it out. A public liquidity insurance scheme implements the second-best because the prospect of a bail-out improves the return-liquidity trade-off. This prevents wasteful self-insurance, boosts long-term investment and market participation which, in turn, feeds liquidity back.

I build a simple three-date model in which ex-ante identical agents invest in long-term projects. They privately observe their project quality at the interim date. These agents also face preference shocks: they can be *normal* or *impatient* and they only learn it ex-interim. At this point, they might want to liquidate a share of their long-term projects either due to private information about future payoffs or because they are impatient. If equilibrium liquidation price is low, there is an ex-ante trade-off between, on the one hand, taking profit of higher output if it is held to maturity and if it succeed and, on the other hand, facing a loss -with respect to storage- in the case of early liquidation.

All other things equal, productivity increases future payoff and thus equilibrium price. If the liquidation price equals the gross return on storage, it does not hurt anymore to liquidate and any date 1 consumption is planed to come from liquidation (storage is wasteful). This is also the price at which *normal* agents with high quality projects enter the market: they start to sell good-quality claims. As all these agents enter the secondary market at the same time, this produces a jump in average quality which accounts for the multiplicity of equilibria. For an intermediate level of productivity and a low price these agents do not participate to the market and equilibrium quality is low. At a higher price however, they participate to the market, which increases average quality and justify this higher price, for the same level of productivity. This is standard result under adverse selection. Clearly, as average quality embodies liquidity, this is such a break that makes liquidity dry-ups possible.

In this model, the market may fail to allocate resource efficiently and expectations about market liquidity have a crucial impact on welfare. Yet, the Government can avoid such a coordination failure by the mean of a costless public liquidity insurance. This result sheds light on the potential welfare losses if the law-maker overlooks this "liquidity expectation channel" when considering public intervention in the case of financial crisis. Caballero and Krishnamurthy (2008) find similar results in a model with Knightian uncertainty.

What is particularly striking is that self-insurance, though individually optimal

[4] The idea is simple and calls back to Akerlof (1970). The market price for a good car in the second-hand market is lower than its intrinsic "fair" value because the average quality of second-hand cars is lower, due to the fact that lemons are more likely to be sold than good cars. Whereas the reservation value is exogenously imposed in Akerlof's paper, Eisfeldt (2004) endogenizes the motivation for trading: current need for resource depends on past decisions and on information about future income.

in a low-liquidity world, is socially costly in two aspects. First because resource are wasted in the storage technology (long-run investment is on average more productive) and second because self-insurance will ex-post prevents agents to provide positive externalities on the secondary market. The fact that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998), and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment was present in Eisfeldt (2004). However, that liquidity dry-ups can endogenously arise for the very reason that investors self-insure against it is a new result[5].

In this model, project quality and shock to preferences are private information. Also, I assume that projects could not be run mutually in the first period and that there is no mean by which agents could credibly commit to invest. Otherwise, agents could form a coalition in order to pool resource and diversify the risk away. This coalition would correspond to the bank in Diamond and Dybvig (1983) and, as there is no aggregate shock to the fundamentals in my model, it could implement a first best allocation.

My assumptions about technology and information structure are maybe too strong to draw definitive conclusions. Nevertheless, it provides new insights on the relationship between liquidity, self-insurance and risk-sharing. In this sense, my results complement the literature on the competing role of banks and market for the provision of Liquidity. Most models share the following features: there is a long term risk-less technology and agents face uncertainty about the timing of their preferred consumption. In Diamond and Dybvig (1983), there is no market and banks can generally provide liquidity and improve on the autarky allocation. Jacklin (1987) shows that this is not the case if there exist a secondary market because the demand deposit contract would not be incentive compatible. Diamond (1987) generalizes this result with a model of exogenous limited market participation. He finds that the lower the participation to the market, to more important the role of the banking sector. In that respect, the key differences of my model are that limited market participation is endogenous and investors are needed to run the initial phase of the project. In such a set-up, the pooled market provides some insurance a bank could not achieve.

Another discrepancy between my work and the banking-liquidity literature lies in the role of idiosyncratic preference shocks. First, while they are at the roots of bank runs in Diamond and Dybvig (1983), it is triking to note that they are not necessary to the existence of liquidity-dry-ups. Second, whereas idiosyncratic shocks are usually a source of adverse selection (Jacklin 1987), they mitigate it in my model and provide insurance. Not only they imply a cross-subsidy to unlucky lemon owners, but also they provide insurance *among* agents that liquidate a good quality project for personal liquidity reasons. The bottom line might appear counter-intuitive, but is simple: the more the idiosyncratic liquidity shocks, the better the market liquidity.

My paper is also related to the corporate finance literature. For instance, Holmström and Tirole (1996, 1998), studied the potential role for the policy maker in providing liquidity to the private sector. In their set-up, the extra-fund the corporate sector might need to complete projects plays the role of preference shocks in the banking literature. One of the major findings is that, absent aggregate shocks, there is no room for public intervention. In my model, this proves not true anymore due to adverse selection.

The remaining of the paper is organized as follows. Section 2 presents a simple model of self-fulfilling liquidity dry-ups. In section 3, I introduce the Government an

[5]See also Gennotte and Leland (1990), Morris and Shin (2004), and Brunnermeier and Pedersen (2009) for model in which liquidity dry-ups are driven by portfolio insurance, loss limits and margin calls respectively.

discuss policy implications . General conditions for multiple equilibria are derived in section 4. Section 5 concludes.

2 A simple Model of Self-fulfilling Liquidity Dry-up

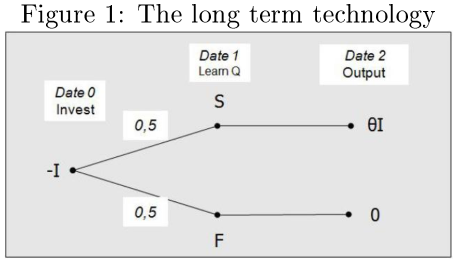
2.1 The economy

Technology and information structure

There are three dates ($t = 0, 1, 2$). At date 0, agents have access to a risky technology. Projects undertaken need two periods to pay off. Additionally, projects might either succeed and yield an output θ per unit invested or fail, in which case they yield nothing. Formally, an amount I invested at $t = 0$ yields, at $t = 2$, an output $IQ_j\theta$ where $Q_j \in \{0, 1\}$ is a dummy reflecting the quality of the project and θ is a productivity parameter common to all projects in the economy. The probability of success is given by $Prob(Q_j = 1) = q > 0$. Investor are needed to initiate the project and have no mean to credibly commit to invest. Moral hazard concerns will consequently restrict ex-ante risk-sharing. Also, projects cannot be physically liquidated at date 1[6]. However, at that date, investors may issue claims to their projects in a competitive pooled market. Claim issuing is assumed to be perfectly anonymous, and for simplicity, the output of the underlying project will be verifiable at date 2. Therefore, I abstract from moral hazard problems once the project has been initiated[7] . Note that there are no other way to borrow against future income than to issue claims on ongoing projects. At dates 0 and 1 , investor have also access to a risk-free storage technology that yields an exogenous rate of return r. There is no depreciation. Finally, to make the analysis interesting, I assume $E[Q_j\theta] > (1 + r)^2$: on average, long-term projects are more productive than storage.

At the beginning of date 1, investor j privately observes his project's quality Q_j. This is private information, and this quality is common to all the projects of a given investor. We can thus think of investors owning only one project of variable size. However, quality is independent across agents. Average quality is thus deterministic. Ex-ante probability of project success is common knowledge.

Figure 1: The long term technology



[6]Equally, I could consider that the physical liquidation costs outweigh the residual value.

[7]Moral hazard does of course play a crucial role in the funding of risky projects (see for instance Stiglitz and Weiss (1981), Kiyotaki and Moore (1997), Holmström and Tirole (1996, 1997 and 1998), and Bernanke, Gertler and Gilchrist (1989)). However, this paper focuses on adverse selection. It makes thus sense to shut down the moral hazard channel in order to identify the consequences of the other side of asymmetries of information.

The problem of the agent

There is a measure one of ex-ante ($t = 0$) identical investors maximizing expected utility, which they derive from consumption at date 1 and 2. At this initial stage, they receive an endowment e which they may allocate between the long term risky investment and the short term storage technologies described in the previous paragraph.

At date 1, agents learn whether they are *normal* or *impatient*. These two kinds of agent differ by the subjective factor β_i they use to discount date 2 utility: $\beta_i \in \{\beta_l, \beta_n\}$ with $0 \leq \beta_l < \beta_n = 1$ and $Prob(\beta_i = \beta_n) = p > 0$.

For simplicity, I will assume *total impatience*, that is: $\beta_l = 0$ which means that agents of this kind do not value utility from consumption at date 2[8]. In the spirit of Diamond and Dybvig (1983), and in line with Eisfeldt (2004) results, *impatient* agents can also be viewed as agent incurring a need for liquidity due to either a current income shock or a very good investment opportunity[9]. As they also learn the quality of their ongoing projects, there are four states of nature. Patience is independent across agents and is also independent of quality. There is thus no aggregate uncertainty in the fundamentals of this model.

Without loss of generality with respect to the central idea of this paper, I assume that period utility is logarithmic, that probabilities are symmetric ($p = 0.5$ and $q = 0.5$) and that endowment and exogenous interest rate are normalized ($e = 1$ and $r = 0$).

Formally, at date 0, agents maximize expected utility as follows:

$$\max_{\lambda, L, S_1} U_0 = E_0 \left[\ln C_1 + \beta_i \ln C_2 \right] \tag{1}$$

$$s.t. \left\{ \begin{array}{c} C_1 + S_1 = 1 - \lambda + LP \\ C_2 = (\lambda - L)\theta Q_j + S_1 \\ 0 \leq \lambda \leq 1 \end{array} \right.$$

Where $E_t[.]$ is the conditional (upon information available at date t) expectation operator[10], C_t is consumption at date t, λ is the share of endowment invested in the long-term technology, $S_1 \geq 0$ is storage between dates 1 and 2 , L is the number of claims (to unit projects) issued at date 1, subject to $0 \leq L \leq \lambda$.

The budget constraints state the following: date 1 resource consist of storage $(1 - \lambda)$ from date 0 plus the revenue from claim issuance (L) at the market price (P). This resource can be consumed (C_1) or transferred to date 2 through storage (S_1). At date 2, the remaining resource are consumed: the output from long-term investment that has not been liquidated $(I - L)\theta Q_j$ plus storage from date 1.

At date 1, conditionally to their type, agents solve an intertemporal consumption problem subject to the budget constraints implied by their date 0 investment decision. They take P, the price at which they may issue claims on their projects (liquidate them) as given. At date 2, all agents consume their remaining resource and die.

2.2 Optimal behavior

Long term investment is risky: on the one hand, it pays very well in the case of success, but it is only valuable for an ex-post *normal* agent. On the other hand, it

[8] Whereas $\beta_n = 1$ is just a normalization, *total impatience* ($\beta_l = 0$) is an extreme case. However, it has no qualitative implication on the main results. See appendix for a discussion.

[9] In Eisfeldt (2004), liquidity shocks are endogenous as they take the form of current income shocks and information about future income shocks that both depend on past investment decisions.

[10] Expectation is thus taken with respect to type ij, that is both patience (β_i) and quality signal on initiated projects (Q_j).

has a relatively low return in any other state of nature. Conversely, storage yields the same amount in each state of nature and is liquid, it can consumed 1 to 1 at each period. There is thus a *return-liquidity* trade-off and risk averse agents might use storage to self-insure against the risk they face. To determine optimal behavior with respect to this trade-off, I solve this problem backward.

Date 1 optimal liquidation policy

Conditionally to the information available at date 1, agents solve a simple intertemporal consumption problem. As the four states of nature are equiprobable, exactly one fourth of agent is of each type. I thus consider four representative date 1 agents, named after their type ij: ns, nf, ls, lf where n stands for *normal* agents, l for *impatient* (or *liquidity hit*) agents, s for success and f for failure. Let $L_{ij}(P, \lambda)$ denote the optimal liquidation correspondence[11] that solves the problem for agent ij for each couple (P, λ).

The first order condition (for $L > 0$) is:

$$\frac{P}{C_1} - \beta_i \frac{\theta Q_j}{C_2} \geq 0 \tag{2}$$

With equality if $L < \lambda$.

Agents that get a failure signal (nf and lf) or get a success signal but is impatient (ls) have no incentive to hold projects to maturity. Formally, as $Q_f = 0$ for the former and $\beta_l = 0$ for the latter, (2) reduces for them to:

$$\frac{P}{C_1} \geq 0$$

Which implies that, as soon as $P > 0$:

$$L_{nf}(P, \lambda) = L_{lf}(P, \lambda) = L_{ls}(P, \lambda) = \lambda \tag{3}$$

They liquidate any project they hold. Note that while the former liquidate because of private information about future payoffs - they want to get rid of their lemons -, the latter do so for idiosyncratic liquidity reasons.

Under log-utility, the liquidation behavior of the remaining agent (ns) is given by:

$$L_{ns}(P, \lambda) = \max(0; \frac{P\lambda - 1 + \lambda}{2P}) \tag{4}$$

The intuition is the following. The optimal liquidation of agent ns is weakly increasing in P and in λ. If both are high enough, $\frac{P\lambda-1+\lambda}{2P}$ is positive because the resource available at date 1 (before liquidation) are smaller than the share of wealth he wants to dedicate to consumption at that period. Conversely, if $\frac{P\lambda-1+\lambda}{2P}$ is negative, the agent would like to "create" ongoing projects. This is of course ruled out by the definition of the long-term technology. In that case, agent ns does not participate in the market and $L_{ns}(P, \lambda) = 0$. Note that if λ is really small, he might roll part of its storage over to date 2.

[11] Note that $L_{ij}(P, \lambda)$ might not be a function. Indeed, an agent ij may reach the same utility level for several values of L. This would for instance happen if $\frac{\theta Q_j}{P} = 1$. However, this will be ruled out in equilibrium.

Date 0 optimal investment policy

Agents choose investment according to expected utility maximization.

The first order condition (for $\lambda > 0$) is:

$$\frac{\partial U_0}{\partial \lambda} = U_{ns}^{'} + U_{ls}^{'} + U_{nf}^{'} + U_{lf}^{'} \geq 0$$

With equality if $\lambda < 1$, and where $U_{ij}^{'} \equiv \left[\frac{\partial U_0}{\partial \lambda} \,|ij\right]$ is the marginal utility of λ conditionally of being in the state ij[12].

*Proposition 1 (**self-insurance**)*

Let $\lambda(P) \equiv \arg\max_{\lambda} U_0 (\lambda, P)$ *be the set of solutions for a given* P *to the date* 0 *problem (1), then:*

$$\begin{cases} \lambda(P > 1) = 1 \\ \lambda(P = 1) = \left[\frac{1}{2}, 1\right] \\ \lambda(P < 1) = \lambda_L \end{cases}$$

With $0 < \lambda_L < \frac{1}{2}$.

This proposition states that when P is smaller than the gross return on storage, investment is also low because it hurts the agent in the states of nature he needs to liquidate. Self-insurance is thus crowding-out productive investment. Conversely, if P is high, investment is high too because it dominates storage - even in the case of early liquidation -, and makes thus the agent better off in all states of nature. If $P = 1$, investment is rather high, though undetermined over the range $(\frac{1}{2}, 1)$ because whereas other agents are indifferent over the whole range of admissible values $(0, 1)$, agent ns is indifferent over this specific range and strictly prefers it to any lower value. This is the reason why the optimal investment policy has not a functional form at $P = 1$.

Proof: see appendix

Supply for claims and average quality

Once I have solved for $\lambda(P)$, I can evaluate the optimal liquidation function (3) and (4) at the optimal investment level given price P (*proposition* **1**):

$$L_{nf} (P, \lambda(P)) = L_{lf} (P, \lambda(P)) = L_{ls} (P, \lambda(P)) = \lambda$$

$$L_{ns} (P, \lambda(P)) = \begin{cases} 0 & ; P < 1 \\ \frac{1}{2} & ; P > 1 \\ \in \left[0, \frac{1}{2}\right] & ; P = 1 \end{cases}$$

And I can define $Q(P)$, the implied average quality of traded claims:

$$Q(P) = \begin{cases} Q_L = \frac{1}{3} & ; P < 1 \\ Q_H = \frac{3}{7} & ; P > 1 \\ Q_1 = \left[\frac{1}{3}, \frac{3}{7}\right] & ; P = 1 \end{cases} \tag{5}$$

With:

[12] As the relevant information is known as of date 1, I could equally have used the following notation: $U_{ij}^{'} \equiv \frac{\partial U_1}{\partial \lambda}$.

$$Q(P) \equiv \frac{\sum\limits_{i,j} L_{ij}\left(P, \lambda(P)\right) Q_j}{\sum\limits_{i,j} L_{ij}\left(P, \lambda(P)\right)} \tag{6}$$

This supply curve gives the average quality one would observe for a given price P. If the price is low, market participation is limited and the proportion of good quality claims is low. However, if the price is high, all agents participates to the market and the average quality is higher.

Demand for claims[13]

There is also a measure m of "buffer" agents. They do not have access to the risky long-term technology. They only have access to storage and to the market for claims to ongoing projects. I assume that they are risk-averse and that their endowment stream and utility function are such that they want to save, for instance for precautionary saving motive, and that they clear the market at the rate r[14]. Alternatively, we could think of this to be a small open economy taking the risk free rate as given.

The average return of a claim is $\frac{Q\theta}{P}$, where θ is the technology parameter common to all projects in the economy. As $Q_f = 0$ and $Q_s = 1$, the average quality of a claim Q is also the proportion of good quality claims in the market. By the law of large numbers, and because quality is independent across investors, the average quality also determines the risk-less return of a perfectly diversified portfolio of these claims (I assume perfect divisibility of claims and also that diversification is costless).

As the buyers are risk averse, they always prefer the diversified portfolio whose return is thus: $1 + \pi \equiv \frac{Q\theta}{P}$. As these agents are net savers and have access to the storage technology, the following no arbitrage condition must hold: $\pi = 0$. Hence, their demand for claims is linear in the quality Q and perfectly elastic with respect to price P:

$$P = Q\theta \tag{7}$$

This implies that the market clearing price will simply be the discounted value of the average payoff.

Endogenous market liquidity

As there are always lemons in the market, equilibrium average quality is strictly bounded above by 1. Consequently, claims to projects that are to succeed are sold (if any) at a discount with respect to θ, the price that would prevail absent asymmetry of information. It is in that sense that adverse selection makes high-quality claims "illiquid". For it determines the illiquidity discount, Q embodies market liquidity in this model. From a market liquidity perspective, it also gives a measure of the proportion of agents trading for other reasons than private information about future payoffs. The next subsection formalizes these equilibrium implications.

2.3 Equilibria

A triple $\gamma \equiv (P^*, \lambda^*, Q^*)$ is an equilibrium for this economy if and only if:

$$\begin{cases} P^* = Q^*\theta \\ \lambda^* \in \lambda(P^*) \\ Q^* = Q(P^*, \lambda^*) \end{cases} \tag{8}$$

[13]These paragraphs are largely inspired by Eisfeldt (2004).

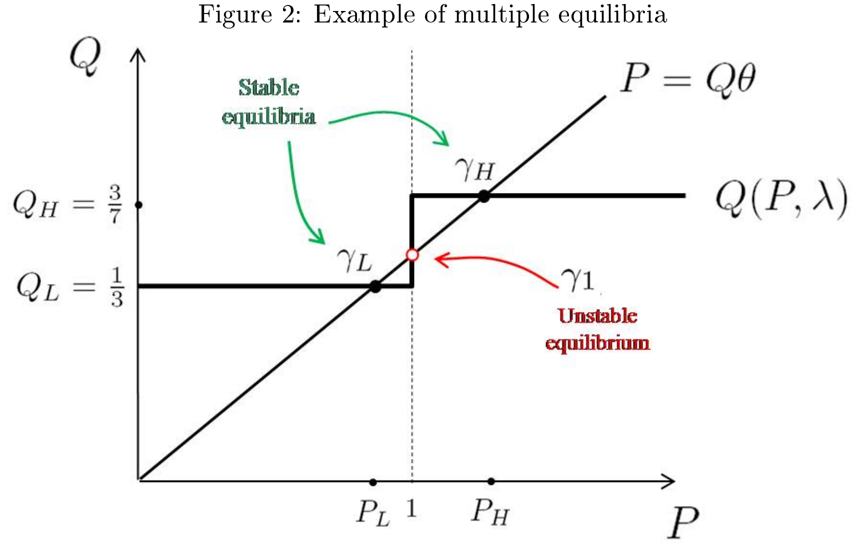
[14]I assume that m is big enough.

That is, P^* is the price buyers are ready to paid for an average quality Q^*, λ^* is an optimal investment decision given P^*, and Q^* is the average quality implied by optimal liquidation behavior at the level of investment λ^* and at price P^*.

There always exist at least one such equilibrium[15]. Uniqueness, however, requires that productivity θ is low enough or sufficiently high. For all the intermediate cases, there are multiple equilibria.

Figure 2 gives an example of multiple equilibria. In a first equilibrium, which I denote $\gamma_L \equiv (P_L, \lambda_L, Q_L)$, agents anticipate a liquidity dry-up. They take as given that the price will be low ($P = P_L < 1$) and, according to *proposition 1,* they choose $\lambda(P_L) < 1/2$. This in turn imply that agents ns will not enter the pooled market at date 1 and that the resulting liquidity will be low: $Q_L = 1/3$ (recall that Q is the proportion of good asset in the pooled market and is thus a direct measure of liquidity). Given Q_L, the market price should be P_L which makes the date 0 expectation a self-fulfilling prophecy: the liquidity has dried up. Similar argument apply to $\gamma_H \equiv (P_H, \lambda = 1, Q_H)$: expecting high liquidity, which means that liquidation does not hurt, agents invest only in the long term technology (because it dominates storage on average). Given that investment is high agent ns enters the market, and its participation increases the proportion of trade for other reasons than private information about future payoff. Hence, equilibrium price and liquidity are indeed high. γ_H is the self-fulfilling high-liquidity and high-investment equilibrium. Both equilibria are locally stable[16] in the sense that agents best-response to any small perturbation to the equilibrium price would bring the price back to equilibrium.

Figure 2: Example of multiple equilibria



[15]Kakutani's theorem ensures that there will always be a price P' such that: $P' \in Q(P', \lambda(P'))\theta$. Such a price obviously pins down an equilibrium, and the corresponding values for Q and λ can be derived from (7) and (6) respectively.

[16]These is also an equilibrium (call it γ_1) which is unstable. In this equilibrium, agents expect $P = 1$. Hence, they are indifferent with respect to investment choice over a wide range of value: $\lambda(P = 1) = (\frac{1}{2}, 1)$. There are several ways to interpret this equilibrium: for instance, it can be seen as a mixed strategy Nash equilibrium, where agents randomize over the two pure strategy investment values $\{\lambda_L; 1\}$, with an expected value $E[\lambda(1)] \equiv \bar{\lambda}_1^* = \frac{(\theta-1)}{2(2\theta-4)}$. Then, the average quality of traded claims is $Q^*(1) \equiv \frac{\bar{\lambda}_1^* + L_{ns}(1, \bar{\lambda}_1^*)}{3\bar{\lambda}_1^* + L_{ns}(1, \bar{\lambda}_1^*)} = \frac{1}{\theta}$ which indeed imply a price $P = 1$. However, in this case, any small perturbation (to the expected price for instance) would switch the best response to either $\lambda^*(P > 1) = 1$ or $\lambda^*(P < 1) < \bar{\lambda}_1^*$. In both cases, best response iteration would not bring the economy back to γ_1.

As productivity increases, the demand curve $P = Q\theta$ rotates clockwise and from a certain threshold ($\theta > 1/Q_L$) there only exists a high-liquidity equilibrium. Conversely, If productivity is low enough ($\theta < 1/Q_H$), only a low-liquidity equilibrium can occur. The third region for productivity, which is formally defined in *proposition 2,* is the most interesting one. As already explained, it corresponds to productivity levels that imply the existence of multiple self-fulfilling equilibria[17].

*Proposition 2 (**dry-ups**)*

If $\Gamma(\theta)$ denote the set of equilibria defined by (8) for parameter θ. Then, $\exists \underline{\theta}, \overline{\theta}$ such that: $\forall \theta \in \left]\underline{\theta}; \overline{\theta}\right[$, $\Gamma(\theta)$ has strictly more than one element.

Proof: Set $\underline{\theta} = 1/Q_H = \frac{7}{3}$ and $\overline{\theta} = 1/Q_L = 3$. From (5) and *proposition* **1** I directly get that $\forall \theta < 3$, $\gamma_L(\theta) = \left(\frac{\theta}{3}, \lambda_L, \frac{1}{3}\right) \in \Gamma(\theta)$ and that $\forall \theta > \frac{7}{3}$, $\gamma_H(\theta) = \left(\frac{3\theta}{7}, 1, \frac{3}{7}\right) \in \Gamma(\theta)$. As these to elements are distinct, I have proposition 2.

The role of productivity θ is straightforward in this simple model because, under log-utility, it only impacts the demand curve. The existence of a productivity range for which there is multiple equilibria does not depend on that assumption and is very robust. Actually, there will always be a discontinuity in the implied quality correspondence if agent ns *(normal* with *success* signal) always chooses to consume at least a little bit at date 1[18]. In section 4, I propose a generalization with respect to p, q, r, e and $u(.)$[19], but here is the main intuition.

All other things equal, productivity increases future payoff and thus equilibrium price. There is also a trigger price at which the return-liquidity trade-off disappears. If the liquidation price equals the gross return on storage ($P = 1$), it does not hurt anymore to liquidate and any date 1 consumption is planned to come from liquidation (investors have long-term horizon and plan to supply for current needs for resource through liquidation). This is also the price at which *normal* (patient) agents with high quality projects enter the market: they start to sell good-quality claims[20]. As all these agents[21] enter the secondary market at the same time, this produces a jump in the implied average quality function. As this trigger price is consistent with two different level of productivity ($Q_H(P = 1)\underline{\theta} = 1 = Q_L(P = 1)\overline{\theta}$) this break accounts for the multiplicity of equilibria. From a market clearing perspective, I have that for an intermediate level of productivity and a low price these agents do not participate to the market and equilibrium quality is low. At a higher price however, they participate to the market, which increases average quality and justify this higher price, for the same level of productivity. This is standard result under adverse selection. Clearly, as average quality embodies liquidity, this is such a break that makes liquidity dry-ups possible.

Market failure, externalities and self-insurance

In this model, the market may fail to allocate resource efficiently. An easy way to prove it is to show that if both exist, the low-liquidity equilibrium is Pareto dominated by the high-liquidity one.

[17] In a similar reduced form set-up, Morris and Shin (2001) shows that if agents receive a signal about productivity, one can solve for a unique equilibrium. In fact, they identify a pivotal value for $E[\theta]$ from which we switch from one equilibrium to the other. In the case productivity is common knowledge, they also have two pure strategy Nash equilibria.

[18] This only requires $u^{'}(0)$ not being too low. For instance, Inada conditions are sufficient but not necessary.

[19] The case of parameter β_l is considered in appendix.

[20] This is the rationale for the here above condition: if it does not hold, it might be the case that these agents never enter the market

[21] Note that they represent a proportion $pq > 0$ of the population.

*Proposition 3 (**market failure**)*

The competitive equilibrium might not be Pareto efficient.

A simple argument, should suffice to convince the reader that all investors[22] are strictly better-off in γ_H than in γ_L. First, any investment held to maturity yields the same amount in both equilibria. Second, the terms of liquidation are better in γ_H , and third, any resource that would have been stored in γ_L yields a higher amount in γ_H whether it is liquidated (and consumed or re-stored) or held to maturity. Therefore, whatever its individual state of the world at date 1, an agent will do better in γ_H. Formal proof: see appendix.

How can it be that everyone is better-off in γ_H? A first clue lies in the issuance of claims to good quality projects. When an agent does so, it increases average quality and all claims can be sold for a better price. There is thus a positive externality. A second piece of evidence is linked to the process of self-insurance. In γ_L, it hurts to liquidate. Accordingly, agents would like to insure against the events that would lead them to do so. As storage is liquid and yields the same amount in every state of nature, they use it as self-insurance and reduce the share of productive investment. What is particularly striking is that such self-insurance, though individually optimal in a low-liquidity world, is socially costly in two aspects. First because resource are wasted in the storage technology (long-run investment is on average more productive) and second because this self-insurance will ex-post prevents agents from providing positive externalities.

The fact that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998) and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment is present in Eisfeldt (2004). However, that liquidity dry-ups can endogenously arise for the very reason that investors self-insure against it is a new result[23]. The fact that market sometimes fail to provide an efficient level of insurance opens the door to government intervention. Before turning to this, I discuss the role of the information structure on equilibrium risk-sharing and the impact of idiosyncratic liquidity shocks on equilibrium liquidity.

Private information and risk-sharing

In γ_H, there is a high level of positive externalities and agents are pretty well insured. Still, they face the risks of being impatient and of receiving the low quality signal, but they do not face anymore the risk to liquidate *in an illiquid market*[24]. However, they are not fully insured, otherwise they would achieve the first best allocation: their marginal utility would be equal across all states of nature. Paradoxically, in γ_L, agents self-insure but they are rather poorly insured. Nevertheless, they still do better than in the absence of market (this case is usually dubbed autarky): If agent could not liquidate at all, it is easy to check that they would self-insure even more (that is $\lambda(0) < \lambda(P_L) = \lambda_L$).

In this model, project quality and shock to preferences are private information. A direct implication is that any contract designed to share the risk on investment would not be incentive compatible. An agent issuing a claim on a bad project would have strong incentives to masquerade an agent with a good project. Also, I

[22] Note that buffer agents are indifferent.

[23] Some ad-hoc frictions are usually used in the literature to generate liquidity crisis. In Gennotte and Leland (1990), portfolio insurance magnifies exogenous shocks. In Morris and Morris and Shin (2004), loss limits might trigger a crisis, and in Brunnermeier and Pedersen (2009), this is margin calls might be destabilizing.

[24] There is empirical evidence that such a risk is priced on financial markets. See Acharya and Pedersen (2005).

assumed that projects could not be run mutually in the first period and that there is no mean by which agents could credibly commit to invest. Otherwise, agents could form a coalition in order to pool resource and diversify the risk away. This coalition would correspond to the bank in the Diamond and Dybvig model, and, as there is no aggregate shocks on the fundamentals of my model, it could implement a first best allocation

These assumptions are quite strong and it might be useful to give a real world example to which the model fit relatively well. Investors might be thought of as financial institutions that have limited resource that they lend to private agents (mortgages or corporate loans for instance). Only these institutions have the needed skills to screen applicants. Screening is assumed costly so that a moral hazard problem prevent them to do it within a principal-agent relationship. However, monitoring is costless and the moral hazard problem does not apply after the investment is indeed committed. The pooled market can then be viewed as a securitization process of the initial loans. All risk is idiosyncratic and is thus diversified away and the buyer of a claim portfolio receive for sure the underlying stream of payment.

Under these assumption, the pooled market provides some insurance a bank could not achieve (recall that a risk sharing deposit contract would not be incentive compatible). This results complement the literature on the competing role of banks and market for the provision of Liquidity. Most models share the following features: there is a long term risk-less technology and agents face uncertainty about the timing of their preferred consumption. In Diamond and Dybvig (1983), there is no market and banks can generally provide liquidity and improve on the autarky allocation. Jacklin (1987) shows that this is not the case if there exist a secondary market because the demand deposit contract would not be incentive compatible. Diamond (1987), generalizes this result with a model of limited market participation. He finds that the lower the participation to the market, to more important the role of the banking sector. Key differences of my model in that respect is that limited market participation is endogenous and investors are needed to run the initial phase of the project.

Idiosyncratic liquidity shocks and adverse selection

In the models cited above, preference shocks are the only source of uncertainty the agents face and, following Jacklin (1987), incentive compatibility problems arise when patient agents pretend they are impatient. In the presence of idiosyncratic productivity shocks there is another source of adverse selection: patient agents want to get rid of their lemons. Whereas private information on preference shocks usually have a negative impact on welfare, it also plays a positive role in my model in the sense that it provides insurance. First, it implies a cross-subsidy from agents with good projects to unlucky lemon owners. Second, they provide insurance *among* agents that liquidate a good quality claim for personal liquidity reasons. The bottom line might appear counter-intuitive, but is simple: the more the idiosyncratic liquidity shocks, the better the market liquidity.

A comparative statics exercise on p, the probability to be patient, may formalize this result. First, it is straightforward to show that $Q_L = \frac{1-p}{2-p}$ and $Q_H = \frac{2-p}{4-p}$ which are both strictly decreasing in p. The higher the proportion of patient agents, the lower average quality. It simply comes from the fact that patient agents always keep at least some of their good quality projects: average quality of claims *conditionally on the seller to be impatient* is strictly higher than average quality of claims (that is $q > Q$). The result is even more striking as $p \to 1$, that is if the probability to incur a idiosyncratic liquidity shock becomes negligible. In that case, average low quality is null ($Q_L = 0$) while average high quality is still positive ($Q_H = \frac{1}{3}$) . The former implies that the price is null ($P = 0$). In effect, the market disappears:

absent personal liquidity shock, everyone that tries to sell must own a worthless lemon. The latter that idiosyncratic shocks are non necessary the model liquidity dry-ups[25]. Furthermore, absent liquidity shocks, equilibrium adverse selection is always worse and, in the case of a dry-up, the market also fails to provide any insurance.

3 Government

In this model and in the case of multiple equilibria, the Government faces a standard second-best problem. Because of positive externalities, it is also standard to find a contingent fiscal policy that rules out bad equilibria[26]. I present in this section a public liquidity insurance scheme that enables the government to achieve it.

3.1 Public Liquidity Insurance

Proposition 4 ***(public insurance)***

The government can implement the second-best.

Formal proof: see appendix.

The following argument is the parallel to Diamond and Dybvig's (1983) deposit insurance with the same assumptions about the fiscal ability of the Government: I assume that the government is able to levy lump-sum taxes and to pay a subsidy contingent on agents behavior. This public insurance scheme should be understood as the promise of a bailout at a minimum price.

Formally, the scheme would consist of a subsidy to liquidation, contingent also to the event of an illiquid market, i.e. a market price P strictly smaller than 1. This subsidy $subs(L, P)$ would be paid proportionally to the quantity of claim issued and would compensate the seller for the loss with respect to storage.

$$subs(L,P) = \begin{cases} (1-P)L & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

To balance its budget, the Government would also levy the following a lump-sum tax:

$$\tau(P) = \begin{cases} (1-P)\sum_{ij} \frac{L_{ij}}{4} & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

Where τ is the per capita lump-sum tax needed to fund the subsidy.

The net effect of such a scheme is thus a transfer from agents that liquidate few to agents that liquidate a lot:

$$transfer_{ij} = \begin{cases} (1-P)\left[L_{ij} - \frac{\sum_{ij} L_{ij}}{4}\right] & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

Under this public liquidity insurance, the ex-ante trade-off between return and liquidity disappears: the date 0 first order condition for $\lambda = 1$ always holds:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

[25] Recall that they are at the roots of bank runs in Diamond and Dybvig (1983).

[26] See Dybvig and Spatt (1983).

Whatever the expected date 1 market price, $\lambda^* = 1$ maximizes expected utility. Hence, it is a dominant strategy to invest the whole endowment in the long-term technology. Under this scheme, the only one equilibrium is the high-liquidity one and the insurance will never be claimed. As in Dybvig and Spatt (1983), such an insurance is thus free. Of course, this result rely on the possibility for the government to levy contingent taxes after observing agents behavior. This is the reason why a private agent could not do it (the reader might want to check that the tax scheme is not incentive compatible). The Government ability to achieve a Pareto improvement *absent aggregate shocks* is at odds with the literature. In Diamond and Dybvig (1983) and Holmström and Tirole (1998), there is a role for the government only in the case of aggregate uncertainty about the fundamentals.

Is it credible, in this model, that the Government commit to bail-out the market in the case it dries-up? The answer is definitely positive. It might be surprising, but as I endowed the Government with the regalian power to raise lump-sum taxes, the only two problems it might face are resource constraint. However, they will never be binding. First because the value of aggregate resource cannot decrease over time. Second, because a highly negative transfer to agent ns would force him to liquidate part of its portfolio. This would trigger positive externalities, increase the liquidation price of all claims, and eventually relax the Government budget constraint before the agent runs out of resource[27].

3.2 Policy implications

During financial crises, the fear of a credit crunch[28] might lead to public interventions such as liquidity injection, bank recapitalization or even nationalization. However, the scope of such public intervention is usually limited by moral hazard concerns. This is indeed well known that public intervention might induce investor to take on too much risk in the future[29] and sow then the seeds of the subsequent crisis.

My results suggest the existence of an additional term to this usual trade-off. Indeed, the public liquidity insurance presented above is equivalent to the promise of a bail-out in case the market is not liquid. Such commitment improves expectations about market liquidity, shifts investors horizon towards the long-run and actually avoids the liquidity dry-up.

These results are highly related to the policy recommendations of Ricardo Caballero[30] about the 2008 financial turmoil. He indeed advocated for not caring so much about moral hazard *during* financial crisis (Financial Times, August 08), and he for instance proposed, in order to launch a virtuous circle, that:

> "The government pledges to buy up to twice the number of shares currently available, at twice some recent average price, five years from now" (VOX, 22 February 2009)

Of course my model abstract too much from moral hazard to draw definitive conclusion on these recommendations, still it provides them some support.

If the government does not want to give that insurance because of moral hazard concerns, it might still try to forbid coordination mechanisms that would lead a liquidity-dry-up. A temporary ban on short-selling might be viewed as such an attempt (see Brunnermeier and Pedersen (2005) for an illustration that short-selling

[27] Where agent ns to liquidate its whole portfolio, the equilibrium price would be $\frac{\theta}{2} > 1$ which is not consistent with a negative transfer to this agent.

[28] See Bernanke, Gertler and Gilchrist (1989) for an illustration of the financial accelerator effect.

[29] See for instance Diamond (1984) and Holmström & Tirole (1997).

[30] See Caballero and Krishnamurthy (2008) for a model in which Knightian uncertainty is the key ingredient that leads to such conclusions.

might coordinate a switch toward a liquidity dry-up). Also, and more generally, lengthening investors horizon might reduce adverse selection on asset markets. This improves liquidity and increases the propensity to invest in long-term risky projects that are on average more productive than short-term investment. If investors horizon is short, such socially profitable opportunities are not concreted and there is a welfare loss. According to that mechanism, the law-maker should avoid policies that shorten investors horizon such as, for instance, mark-to-market accounting. Finally, transaction taxes, which have been considered as a mean to reduce speculation and promote long-term investment (see for instance Tobin (1974) and Stiglitz (1989)) might in fact prove counter-productive: it would increase self-insurance and reduce liquidity. This, in turn, would reduce the propensity to invest in long-term projects.

4 General conditions for multiple equilibria

Multiplicity depends on parameter values. There is a in fact a broad range of such values for which there exists multiple equilibria. In this subsection, I establish it rigorously: I show that for any admissible value for parameters $\{p, q, e, r\}$, there exist corresponding values of the remaining parameter, namely θ, that ensure the existence of at least two stable[31] equilibria, under mild assumption about preferences[32].

let define the generalized implied price correspondence $P'(P,\theta) : \left(0, \frac{\theta}{(1+r)}\right) \to \left(0, \frac{\theta}{(1+r)}\right)$:

$$P'(P,\theta) = \begin{cases} P'_L(P,\theta) = \frac{q-pq}{1-pq}\frac{\theta}{(1+r)} & ; P \leq 1+r \\ P'_H(P,\theta) = \left[\frac{q-pq\lambda^*_{ns}(P,e,r,\theta)}{1-pq\lambda^*_{ns}(P,e,r,\theta)}\right]\frac{\theta}{(1+r)} & ; P \geq 1+r \\ P'(1+r,\theta) \in \left[0, \frac{\theta}{(1+r)}\right] & ; P = 1+r \end{cases}$$

With $\lambda^*_{ns}(P,e,r,\theta)$ being the solution to (**??**) generalized with respect to e and r, but restricted to the case $P \geq 1+r$[33].

Before turning to *proposition 3*, I establish the continuity of $P'_H(P,\theta)$ and the fact that liquidity is always higher when agent ns participates the market.

Lemma 1 (boundedness)

Let $Q_i(P,\theta) \equiv P'_i(P,\theta)\frac{(1+r)}{\theta}$ *be the level of liquidity implied by* (P,θ)*, then:*

$$Q_L(\theta) \leq Q_H(P,\theta) \leq q$$

Proof: *Lemma 1* derives directly from the fact: $\lambda^*_{ns} \in [0,1]$.

Lemma 2 (continuity)

$P'_H(P,\theta)$ *is continuous in* P *and* θ*.*

[31]As illustrated in *figure* 2, there is also one class of unstable equilibria, which I will not discuss further here.

[32]Furthermore, as the reader might easily check, it could also be done other ways around: given all other parameters, one can solve for the range of p, q or r that generates multiple equilibria. Note that it is not the case for parameter e.

[33]$\lambda^*_{ns}(P,e,r,\theta) = \arg\max_\lambda u(e(1-\lambda)P) + u(e\lambda\theta)$

Proof: As $\lambda_{ns}^* \in [0,1]$, $P_H^{'}$ is a continuous function of $\lambda_{ns}^*(P,e,r,\theta)$. The implicit functions theorem applied to the first order condition for an interior λ_{ns}^* ensures that $\lambda_{ns}^*(P,\theta)$ is continuous in both its arguments which implies *Lemma 2*.

Condition 1: $u^{'}(0) > u^{'}\left(e\frac{(1+r)^2}{q}\right)\frac{1-pq}{q-pq}(1+r)$

This (mild) condition will ensure $C_{1,ns}^* > 0$, that is I rule out the case where agent ns optimally chooses not to consume at date 1.

Lemma 3 (liquidity dominance)

Under *condition **1***, $Q_L(\theta) < Q_H(1+r,\theta)$.

Proof:

As $\frac{(1+r)^2}{q}$ is the lower bound for θ (risky projects are on average more productive than storage) and $\frac{1-pq}{q-pq} = Q_L(\theta)^{-1}$ is an upper bound for $Q_H(P,\theta)^{-1}$, given Lemma 1, *Condition 1* implies that: $u^{'}(0) > u^{'}(e\theta)\frac{\theta}{P_H}$. With $P_H = \frac{Q_H(P,\theta)\theta}{(1+r)}$. This, in turn, implies that $\lambda_{ns}^*(1+r,e,r,\theta) > 1$ which implies *Lemma* 3.

*Proposition 5 (**multiplicity - general case**)*

Let $\bar{\Omega}$ be the range of admissible values[34] *for parameters$\{p,q,e,r\}$. Let $\Gamma(\Omega,\theta) = \{P^*,\lambda^*\}$ denote the set of stable equilibria defined by (8) for a vector $(\Omega \in \bar{\Omega},\theta)$ of parameters. Assume $u'(0) > u'(e\theta)\frac{\theta}{P}$.*

$\forall\Omega \in \bar{\Omega}$, $\exists\underline{\theta} < \overline{\theta}$ such that $\forall\theta \in]\underline{\theta};\overline{\theta}[$, $\Gamma(\Omega,\theta)$ has at least two elements;

Proof: As I only consider stable equilibria, I am not interested in the vertical locus ($P = 1+r$). I will consider separately the two functions $P_L^{'}(P,\theta)$ and $P_H^{'}(P,\theta)$ defined respectively on the sets $(0,1+r)$ and $(1+r,\frac{\theta}{(1+r)})$ and show that there exists a range of θ that generates at least an equilibrium for both functions:

The upper bound for a low-liquidity equilibrium Brouwer's fixed point theorem gives the necessary and sufficient condition on θ for a unique fixed point $P_L^{'}(P,\theta) = P$:

$\exists P \in [0,1+r]$ such that $P_L^{'}(P,\theta) = P \iff \theta \leq (1+r)^2\left[\frac{1-pq}{q-pq}\right]$.

There exists thus a unique low-liquidity equilibrium if and only if θ is low enough, and I can thus set:

$$\overline{\theta} \equiv (1+r)^2\left[\frac{1-pq}{q-pq}\right] = \frac{(1+r)^2}{Q_L(\overline{\theta})}$$

The lower bound for a high-liquidity equilibrium: In order to derive a sufficient condition for the existence of a fixed point $P_H^{'}(P,\theta) = P$, I construct a function $G(P,\theta) \equiv P - P_H^{'}(P,\theta)$, defined on the interval: $\left[1+r,\frac{\theta}{(1+r)}\right]$. Clearly, given continuity $P_H^{'}(P,\theta)$ (*Lemma 2*) of , if $G(P,\theta)$ changes sign on its domain, there is a fixed point for $P_H^{'}(P,\theta)$.

Since $P_H^{'}(P,\theta) = \frac{Q^*(P,\theta)\theta}{(1+r)}$ and $Q^*(P,\theta)$ is bounded above by $q < 1$, I have: $P_H^{'}(P,\theta) < \frac{\theta}{(1+r)}$ and thus $P_H^{'}\left(\frac{\theta}{(1+r)},\theta\right) < \frac{\theta}{(1+r)}$. It implies:

$$G\left(\frac{\theta}{(1+r)},\theta\right) > 0 \tag{9}$$

[34] $0 < p < 1$, $0 < q < 1$, $e > 0$, $r > -1$

For any θ, given (9), a sufficient condition for the existence of a high-liquidity equilibrium is thus:

$$G\left((1+r),\theta\right) \leq 0$$

Which is equivalent to:

$$P_H^{'}(1+r,\theta) \geq (1+r)$$

And thus to:

$$\theta \geq \frac{(1+r)^2}{Q_H(1+r,\theta)} \tag{10}$$

As $Q_H(P,\theta)$is bounded, there will always exist a θ high enough such that condition (10) is satisfied. However, I am interested in a lower bound on θ for this condition to hold. That is, a $\underline{\theta}$ such that:

$$\forall\theta \geq \underline{\theta},\ \theta \geq \frac{(1+r)^2}{Q_H(1+r,\theta)}$$

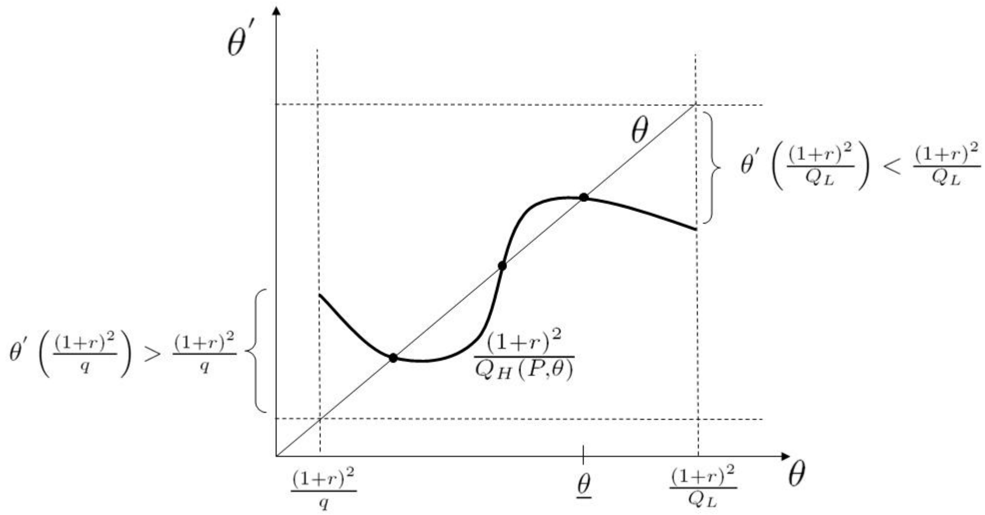
In order to find $\underline{\theta}$, I construct the function $\theta^{'}(\theta) \equiv \frac{(1+r)^2}{Q_H(1+r,\theta)}$. Given boundedness of $Q_H(P,\theta)$ (*Lemma* 1), it is bounded below by $\frac{(1+r)^2}{q}$ and above by $\frac{(1+r)^2}{Q_L(\theta)}$. Also, given *Lemma* 2, it is continuous over the corresponding range: $\left[\frac{(1+r)^2}{q}, \frac{(1+r)^2}{Q_L(\theta)}\right]$. It admits thus at least a fixed point: $\theta^{'} = \frac{(1+r)^2}{Q_H(1+r,\theta^{'})}$.

For a wide class of utility functions[35], this fixed point is unique, and *Lemma 1* $Q_H > Q_L$ implies that $\theta^{'}(\frac{(1+r)^2}{Q_L(\theta)}) < \frac{(1+r)^2}{Q_L(\theta)}$ and thus $\forall\theta \geq \theta^{'}$, $\frac{(1+r)^2}{Q_H(1+r,\theta)} < \theta$. Hence, this unique fixed point is a lower bound on θ for the existence of a high-liquidity equilibrium. I can thus choose $\underline{\theta} \equiv \theta^{'}$. If there are multiple fixed points, the correct lower bound is the highest valued fixed point:

$$\underline{\theta} \equiv \max\left\{\theta^{'} : \theta^{'} = \frac{(1+r)^2}{Q_H(1+r,\theta^{'})}\right\}$$

[35]Including CARA, CRRA and quadratic utility functions.

Figure 3: Fixed point $\theta^{'} = \theta$. Example with multiple solutions.



The range for multiple equilibria *Lemma 3* implies $\underline{\theta} < \overline{\theta}$ which concludes the proof: $\forall\theta \in]\underline{\theta};\overline{\theta}[$ there exists at least two equilibria.

5 Concluding comments

In this paper, I focused on self-fulfilling liquidity dry-ups and I showed the major role adverse selection plays in such phenomena. The fact that the government can improve on the resource allocation, even in the absence of aggregate shock is at odds with the banking literature. Also, the result that promise of a bail-out implements the second-best is important. Indeed, most of the debates on public intervention during financial crisis focus on moral hazard, and adverse selection has been rather overlooked.

While my model abstract too much from moral hazard to draw definitive conclusion it still shed some new light on financial crisis. In normal times, the feedback effect between liquidity and investment magnifies the externalities linked to the issuance of good quality claim for consumption reallocation motive. However, the anticipation of a dry-up suffice to reverse the process: the feedback effect magnifies the negative externalities linked to self-insurance.

This mechanism seems powerful but the model does not say a word on magnitude of the potential welfare gain in comparison with welfare losses usually attributed to moral hazard. Still, I suspect that the introduction of moral hazard would not wipe out the results and that it is worth extending the model in that direction. This view is based on the fact, even in the high-liquidity equilibrium, agents are only partially insured. There is thus still room for incentive to exert effort. On the other hand, aggregate shocks to preferences or productivity might also have a negative impact on the government budget constraint and the public liquidity insurance would probably need to be more sophisticated to ensure that this constraint is satisfied in all states of nature.

6 Appendix

6.1 Proof of proposition 1 (self-insurance)

1. $\lambda(P > 1) = 1$
2. $\lambda(P = 1) = \left(\frac{1}{2}, 1\right)$
3. $\lambda(P < 1) = \lambda_L$ with $0 < \lambda_L < \frac{1}{2}$

First

- $U'_{ls} = U'_{lf} = U'_{nf} = (P-1)u'(C_1)$ whose sign is the one of $(P-1)$
- $U'_{-nf} = U'_{-nf} = \frac{(1-P)}{1-\lambda}$
- $U'_{ls} = \frac{2(1-P)}{1-\lambda}$

The study of the sign of U'_{ns} is a bit more complicated

- Formally:
- $U'_{ns} = -\frac{1}{1-\lambda+PL^*} + \frac{1}{\lambda-L^*}$
- I have also:

The first order condition for $L^*_{ns} = 0$ is $u'(1-\lambda)P \leq u'(\lambda\theta)\theta$ or $-\frac{P}{1-\lambda} + \frac{1}{\lambda} \leq 0$ which reduces to $\lambda \leq \frac{1}{1+P}$

- Assume $L^*_{ns} = 0$ then $U'_{ns} = -\frac{1}{1-\lambda} + \frac{1}{\lambda} \gtreqless 0$ iif $\lambda \lesseqgtr 1/2$
- Thus if $P \gtreqless 1$ then $\lambda \lesseqgtr 1/2$ and $U'_{ns} \gtreqless 0$

The first order condition for $L^*_{ns} > 0$ is $u'(1-\lambda+LP)P = u'((\lambda-L)\theta)\,\theta$ which reduces to $L = \frac{P\lambda-1+\lambda}{2P}$

- Assume $L^*_{ns} > 0$ then $U'_{ns} \gtreqless 0$ iif $\lambda \gtreqless \frac{1}{1-P}$ that is, if $P \gtreqless 1$

So,

- If $P > 1$ and then $U'_{ns} > 0$
- If $P < 1$ and $\lambda > 1/2$ then $U'_{ns} < 0$
- If $P < 1$ and $\lambda < 1/2$ then $U'_{ns} > 0$
- If $P = 1$ and $\lambda \geqslant 1/2$ then $U'_{ns} = 0$
- If $P = 1$ and $\lambda < 1/2$ then $U'_{ns} > 0$

Summary

- If $P > 1 \Rightarrow U'_{ij} > 0 \; \forall ij, \lambda$ which **gives 1.**
- If $P = 1 \Rightarrow U'_{ij} \geq 0 \; \forall ij, \lambda$ and $U'_{ns} > 0$ if $\lambda < 1/2$ which **gives 2.**
- If $P < 1$ and $\lambda > 1/2 \Rightarrow U'_{ij} < 0 \; \forall ij$
- If $P < 1$ and $\lambda = 1/2 \Rightarrow U'_{ns} = 0 \; \forall ij$ and $U'_{ij} < 0 \; \forall ij \neq ns$
- If $P < 1$ and $\lambda > 0 \Rightarrow U'_{ns} > 0 \; \forall ij$ and $U'_{ij} < 0 \; \forall ij \neq ns$
- The last three, as $u(.)$is increasing and concave **give 3.**

6.2 Proof of proposition 3 (market failure)

The competitive equilibrium might not be Pareto efficient.

Let $\theta \in \left]\frac{7}{3};3\right[$ and $\Gamma(\theta) = \{\gamma_L;\gamma_H\}$ with $\gamma_L = \left(\frac{\theta}{3},\lambda_L,\frac{1}{3}\right)$ *and* $\gamma_H = \left(\frac{3\theta}{7},1,\frac{3}{7}\right)$ or $\Gamma(\theta) = \left\{\gamma_L = \left(\frac{\theta}{3},\lambda_L,\frac{1}{3}\right);\gamma_H = \left(\frac{3\theta}{7},1,\frac{3}{7}\right)\right\}$.

Denote C_t^{γ} the optimal consumption of the considered agent at date t in equilibrium γ.

If all agents are better-off in γ_H, it Pareto dominates γ_L.

- Impatient agents (ls and lf) are better-off

Obvious since $C_1^{\gamma_L} = P_L < P_H = C_1^{\gamma_H}$ and $C_2^{\gamma_L} = C_2^{\gamma_H} = 0$.

- Impatient nf is better-off

Obvious since $C_1^{\gamma_L} = C_2^{\gamma_L} = \frac{P_L}{2} < \frac{P_H}{2} = C_1^{\gamma_H} = C_2^{\gamma_H}$

- Impatient ns is better-off

If S_1^*denotes the optimal level of savings of this agent at date 1, I have:

$$\begin{cases} C_1^{\gamma_L} = 1 - \lambda_L - S_1^* \\ C_2^{\gamma_L} = \lambda_L\theta + S_1^* \end{cases}$$

In γ_H, the budget constraints of this agent are:

$$\begin{cases} C_1 \leq P_H L \\ C_2 \leq (1-L)\theta \end{cases}$$

- Assume he sets: $L = \frac{C_{1ns}^{\gamma_L}}{P_H}$, then:

$$\begin{cases} C_{1ns}^{\gamma_H} = C_{1ns}^{\gamma_L} \\ C_{2ns}^{\gamma_H} = \left(1 - C_{1ns}^{\gamma_L}\right)\theta \end{cases}$$

and $C_{2ns}^{\gamma_H} > C_{2ns}^{\gamma_L}$ as $\lambda_L < 1$ and $S_1^* \geq 0$. He can thus do better than in γ_L, which concludes the proof.

6.3 Proof of proposition 4 (public liquidity insurance)

Under this liquidity insurance, date 1 budget constraints are then contingent to P:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + L\max(P,1) - \tau(P) \\ C_2 = (\lambda - L)\theta Q_j + S_1 \end{cases}$$

Where:

$$\tau = \begin{cases} (1-P)\sum_{ij}\frac{L_{ij}}{4} & ;P < 1 \\ 0 & ;P \geq 1 \end{cases}$$

It simply states that if the market liquidation price is low, agents will have to pay $\tau(P)$ but they will also be compensated for the loss of value with respect to the opportunity cost - the return on storage.

Of course, I still have: $L_{nf}^*(P) = L_{ls}^*(P) = L_{lf}^*(P) = \lambda^*$. Such a subsidy will not decrease the willing to liquidate of these agents.

Thus:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + \lambda\max(P,1) - \tau(P) \\ C_2 = (\lambda - L)\theta Q_j + S_1 \end{cases}$$

Conditionally on "not being ns", the date 0 first order condition for ($\lambda = 1$) always holds:

$$E_0 \left[\max(P-1,0) \ln^{'}(C_1) + \beta_j \ln^{'}(C_2)\theta Q_j \, |ij \neq ns \right] > 0 \qquad (11)$$

The return-liquidity trade-off has well disappeared and as it is true irrespectively to the competitive market price, $\lambda = 1$ is a dominant strategy.

Agent ns:

If $P \geq 1$:

$$\begin{cases} C_1 = 1 - \lambda + \max(\lambda - 1/2; 0)P \\ C_2 = (\lambda - \max(\lambda - 1/2; 0))\theta \end{cases}$$

$\lambda > 1/2 => L_{ns} = \lambda - 1/2 => (P-1)\ln^{'}(C_1) > 0$
$\lambda < 1/2 => L_{ns} = 0 => -\ln^{'}(1-\lambda) + \ln^{'}(\lambda\theta)\theta > 0$

If $P < 1$:

$$\begin{cases} C_1 = 1 - \lambda + L - \tau \\ C_2 = (\lambda - L)\theta \end{cases}$$

$\lambda > \frac{1-\tau}{2} => L_{ns} = \lambda - \frac{1-\tau}{2} => FOC = 0$
$\lambda < \frac{1-\tau}{2} => L_{ns} = 0 => -\ln^{'}(1-\lambda-\tau) + \ln^{'}(\lambda\theta)\theta > 0$

Hence:

$$\frac{\partial U_{ns}(\lambda, P)}{\partial \lambda} \geq 0 \qquad (12)$$

and (11) plus (12) gives:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

Which concludes the proof.

6.4 Partial impatience

Proposition 5 established the robustness of the main result with respect to most parameters of the model. Here I show robustness with respect to a strong assumption, namely total impatience ($\beta_l = 0$).

In fact, $\beta_l > 0$ has an impact on contingent behavior.

- First, and by definition, there is no impact at all on agents ns and nf.
- Second, the impact on lf is irrelevant with respect to this analysis: there is no impact on liquidation behavior ($L^*_{lf}(P) = \lambda^*$) and the impact on S^*_1 has no effect on liquidity.
- Third, there is an impact on ls behavior: he does not liquidate his whole portfolio as before and this has an impact on market liquidity.

For the ease of exposure, I pick up the following parameters: $p = q = 0.5$, $e = 1$ and $r = 0$ and

Agent ls solves:

$$\max_{L,S_1} \ln(C_1) + \beta_L \ln(C_2)$$

$$s.t. \begin{cases} C_1 + S_1 = (1-\lambda) + LP \\ C_2 = (\lambda - L)\theta + S_1 \\ 0 \leq L \leq \lambda \\ S_1 \geq 0 \end{cases}$$

The first order conditions are:

$$\begin{cases} \frac{P}{C_1} - \beta_L \frac{\theta}{C_2} \leq 0; \quad (=0)\; if\; L > 0 \\ \frac{1}{C_1} - \beta_L \frac{\theta}{C_2} \leq 0; \quad (=0)\; if\; S_1 > 0 \end{cases}$$

Thus:
If $P \leq \frac{\beta_L(1-\lambda)}{\lambda}$ => $L^*_{ls} = 0$ (and so is L^*_{ns}, of course) => $Q^* = Q_L = 1/3$.
If $\frac{\beta_L(1-\lambda)}{\lambda} < P < 1$=> $L^*_{ls} > 0$

That is, if P is high enough, agents ls starts to issue claims on good quality projects and liquidity increases.

If $P \geq 1$, we are back in the model of *section* 2 with the difference that now $L^*_{ls} = \lambda^* - \lambda^*_{ls}$ instead of $L^*_{ls} = \lambda^*$. This has an effect on the level of quality but not on the existence of a break at $P = 1$. There will thus still be θ's for which there are multiple equilibria.

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