Voting on pensions: sex and marriage.

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Abstract

Existing political economy models on pensions focus on age and productivity. In this paper we focus on two other individual characteristics: sex and marital status. We assume away age (people vote at the start of their life) and thus look at the most preferred rate of taxation that finances a Beveridgean pension of individuals that are characterised by a certain wage rate, sex and marital status. Marriage pools both wage and longevity differences between men and women. Hence singles tend to have more extreme most preferred tax rates. The majority voting solution will thus depends on wage and longevity differences but also on the relative number of couples versus singles.

Ketwords:social security, differential longevity, voting on pensions

1 Introduction

If there were no limit to the length of a title, we would have entitled our paper: Why do men consistently agree with pension schemes that penalize them? One indeed knows that most pension systems provide benefits that are longevity-invariant and sometimes contribution-invariant. Given that men have a shorter life expectancy than women and earn and thus contribute more than women, it is clear that such pension systems are to their detriment. The first answer one can offer to this question is trivial: women outnumber men and thus can impose their view. Another answer is that with flat rate benefits low income men can back such schemes granted that earnings differences dominate longevity differences. Yet the best answer might be that in a society

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where a majority of men and women are married longevity and earnings differences are pooled within the couple and this makes any sex war irrelevant.

Women live longer than men and they earn less than men on average. At the same time, one also knows that low income people, men and women, have a lower longevity than high income people. This has lead to studies showing that social security systems that look redistributive but provide longevity-invariant benefits are in fact not that redistributive (see e.g. Coronado et al. 2000, Liebman 2001 and Bommier et al. 2006). Taking just one example, Bommier et al. (2006) estimate that the French public pension system redistribution is reduced up to 50% because it longevity invariant.

There exist a number of political economy papers trying to explain existing pension systems by majority voting.[1] The first one by Browning (1985) focuses on the age differences: the old being in favour of generous pensions and the young preferring private saving, in that kind of model, the decisive voter is the median age one. A second generation of models introduces differences not only in age but also in wage rates. For example, Casamatta et al. (2002) show that the pension system is chosen by a majority made of rich and poor workers who collude against a coalition of retirees and middle class workers; this is the so-called *ends against the middle* outcome.

In this paper we want to focus on the majority chosen pensions in a society where a majority of men and women are married and a minority is single. Men live shorter and earn more than women. Assuming that retirement consumption is financed by the proceeds of private saving and by a Beveridgean pension, we want to test several hypotheses:

- what is the effect of longevity and wage gender gap on the chosen tax rate?
- what is the effect of increasing the number of married couples on the generosity of pensions?
- what is the effect of increasing the relative number of two-breadwinners (versus one-breadwinners) couples on the level of benefits?
- what is the incidence of adjusting the couples' pension benefits for scale economies?
- how the above results are affected by introducing the idea that single men longevity is quite lower than that of married men?

[1] For a good survey, see de Walque (2005).

The setting we adopt is standard. People live for two periods, work in the first and retire in the second. They control two variables: their private saving and the payroll tax rate through voting. To keep the analysis tractable we make a number of assumptions: quasi linear utility function; same density function for the wages of women and men, the first ones being a fixed fraction of the second; men marry women who have the same wage as theirs up to that fraction; all men have the same longevity that is lower than that of women; no liquidity constraints; uniform pension benefits (Beveridgean system) and uniform payroll tax rate.

In this paper, we answer the above questions using a political economy argument. We first consider a society where all men and women are single and look at the effect of increasing the wage or the longevity gap on the majority voted tax rate. We then compare a society in which there are only couples to a society where there are only singles. We show that the existence of couples in a society neutralizes gender differences in longevity and in productivity while in a society of only singles, the level of the tax rate depends on gender gaps in longevity and in productivity. We obtain that the size of the pension system will be greater in a society with only couples, if differences in productivity are small. On the contrary, for reasonable levels of differences in longevity and in productivity, the tax rate is likely to be higher in a society with only singles. We also extend our analysis to a "mixed society" in which we allow for the coexistence of single men, single women and couples. Our results go in the same direction; if there are only differences in longevity between men and women, an increase in the number of couples will increase the size of the pension system. On the contrary, if there are only differences in wages, the result is ambiguous. When we introduce the idea that a fraction of couples consist of one-breadwinners we show that the equilibrium tax rate increases or decreases with that fraction depending on whether the couple of one-breadwinners get together two or one pension.

2 The basic model

In this model, we assume that individuals live at most for two periods. Each individual works in the first period and retires in the second period. Each individual of type i is characterised by a pair (w_i, π_i) where w_i is the labour productivity in the first period and π_i is the length of the second period of life.

The intertemporal utility function of any individual of type i is quasi-linear in the first period consumption and is represented by

$$u_i\left(c_i, d_i, l_i\right) = c_i - v\left(l_i\right) + \pi_i u\left(d_i\right)$$

where c_i and d_i denote the first and second period consumptions respectively and l_i is the labour supply in the first period. Second-period utility function, $u(.)$ is such that $u'(.) > 0$ and $u''(.) < 0$. The disutility of labor, $v(l_i)$ is quadratic and equal to $l_i^2/2$ so that it increasing and strictly convex. In our model, individuals supply labour, contribute to the pension system, consume and save in the first period. In the second period, they retire and receive a pension benefit p. We also assume a perfect annuity market so that the return on savings is simply $1/\pi_i$. First and second period consumptions can then be written as

$$\begin{aligned} c_i &= (1-\tau) w_i l_i - s_i \\ d_i &= \frac{s_i}{\pi_i} + p \end{aligned}$$

where $\tau \in [0,1]$ is the payroll tax rate and s_i is the amount of savings.

Throughout the paper, we assume away liquidity constraints so that s_i can be positive as well as negative. The problem of this individual consists in solving

$$\max c_i - l_i^2/2 + \pi_i u(d_i)$$
$$\text{s.to} \begin{cases} c_i = (1-\tau) w_i l_i - s_i \\ d_i = \frac{s_i}{\pi_i} + p \end{cases}$$

From the first order conditions of this problem we obtain:

$$\begin{aligned} l_i^* &= (1-\tau) w_i \\ u'(d_i^*) &= 1 \end{aligned}$$

As to the pension system, we assume that individuals contribute to the pension system during the first period of their life and receive a flat pension benefit in the second period of their life (i.e. the retirement period), which has length π_i for any agent i. Thus a feasible pension system must satisfy the following budget constraint,

$$p \sum n_i \pi_i \leq \sum \tau n_i w_i l_i^*$$

where n_i denote the relative number of individuals of type i. Note that here, p is an annual pension benefit which implies that a person who lives long gets in total more than a person who has a short life. Under the assumption of perfect budget balance, we obtain the expression of the pension benefit,

$$p(\tau) = \tau \frac{(1-\tau) Ew^2}{\bar{\pi}}$$

where we use the expectation operators. Every individual contributes an amount which is proportional to his labour income and receive a uniform pension benefit during a retirement period of unequal length. Such a pension system thus operates income redistribution from high- productivity individuals toward low-productivity ones and from short-lived individuals toward long-lived ones.[2]

We further define the indirect utility function of an individual of type i as

$$V^i(\tau) = \frac{(1-\tau)^2 w_i^2}{2} - s_i^* + \pi_i u\left(\frac{s_i^*}{\pi_i} + p(\tau)\right)$$

The preferred tax rates of this individual is obtained by solving the following program:

$$\max_{\tau\in[0,1]} V^i(\tau)$$

In appendix, we show that the preferred tax rate level is

$$\tau^{i*} = \begin{cases} 0 \text{ for } \frac{w_i^2}{Ew^2} \geqslant \frac{\pi_i}{\bar{\pi}} \\ \frac{\frac{\pi_i}{\bar{\pi}}Ew^2 - w_i^2}{\frac{2\pi_i}{\bar{\pi}}Ew^2 - w_i^2} \text{ otherwise} \end{cases} \tag{1}$$

The level of the tax rate chosen by the individual depends on the level of expected redistribution he gets from the pension system. There are two possible ways he may gain from the pension system: either because he has a longer life duration than the average one or because, he has a lower productivity than the average one. Hence, the preferred tax rate of any individual will be zero if he has characteristics such that $w_i^2/\pi_i \geqslant Ew^2/\bar{\pi}$. It is clear that the lower the wage rate and the higher the longevity the likelier an individual will be in favor of the pension scheme. The equality $w_i^2/\pi_i = Ew^2/\bar{\pi}$ gives us the separating locus of types which divides those who are in favor and those who are against the tax. In the plane (w, π) this locus is decreasing: to its left, one finds the types who are in favor of a positive tax; to the right, they are against. One can also note that, when positive, the most preferred tax decreases with w and increases with π. Take the case of someone with a zero wage, one has that his most preferred tax rate is equal to 1/2 and not 1. This comes from the efficiency cost of taxation: 1/2 is the tax that provides the maximum revenue, i.e. the peak of the Laffer curve.

Majority voting when there are two characteristics raises some technical problems that will be solved below by assuming a particular relation between the two characteristics. For the time being assume that all individuals have the same longevity $\pi_i = \bar{\pi}$ and that the wage rate have the standard density

[2] Most PAYG pension schemes exhibit such features.

function with median wage below average wage: $\bar{w} \geq w_m$. We also know from Jensen inequality that $\sqrt{Ew^2} > \bar{\pi}$. Given that in the relevant range of w, the most preferred tax decreases with w, the Condorcet winners are the individuals with a median wage.

3 Singles only.

3.1 Assumptions

Up to now we have spoken of types and we have not yet introduced genders and marriage. We start by the genders and make a number of assumptions. Our society comprises the same number of men and women. These are characterised by a pair (π, w) for men and by a pair (π_f, w_f) for women such that

$$\begin{aligned} \pi_f &= \beta\pi \\ w_f &= \alpha w \end{aligned}$$

with $\alpha \leq 1$ and $\beta \geq 1$. In other words we posit that women always have a higher life duration than men but also face lower wage.[3] Longevity takes just two values which is not the case of wages. We assume that w follows a uniform distribution, with support $[0, 1]$. The average and the median productivities are then identical and equal to $\bar{w} = w_m = 1/2$. This implies that w_f is distributed over $[0, \alpha]$ with density, $1/\alpha$.

For the time being we only have singles. With these assumptions we can write the pension benefit formula from the revenue constraint. It is the same as in the previous section[4]

$$p(\tau) = \tau \frac{(1-\tau)(1+\alpha^2)}{3\pi(1+\beta)} \tag{2}$$

where $\bar{\pi} = \pi(1+\beta)$ and $Ew^2 = (1+\alpha^2)/3$ with a uniform density. At this point of the analysis, it is also important to stress that the pension benefit is increasing with α and decreasing with β. A drop in α implies a decrease in women contributions so that pension benefit has to decrease to ensure budget balance. On the contrary, an increase in β leads to a decrease in the level of the pension benefit because it implies a longer retirement period for women.

[3]For simplicity, we restrict attention to the most realistic case, where $\alpha \leq 1$ and $\beta \geq 1$ but the analysis could be extended to $\alpha > 1$ and $\beta < 1$.

[4]Later in the paper, we will come back on the case of a one-breadwinner couple. In this case, the problem is slightly different as the pension system budget constraint is modified.

3.2 Preferred tax rates of singles

Under these new specifications we can easily compute the most preferred tax rates of *single men and single women*. Using (1) and replacing for (π, w) in the case of a man and by $(\beta\pi, \alpha w)$ in case of a woman, we get

$$\tau^M(w) = \begin{cases} 0 \text{ for } w^2 \geqslant \frac{(1+\alpha^2)}{3(1+\beta)} \\ \frac{\frac{(1+\alpha^2)}{3(1+\beta)} - w^2}{2\frac{(1+\alpha^2)}{3(1+\beta)} - w^2} \text{ otherwise} \end{cases} \tag{3}$$

$$\tau^F(w) = \begin{cases} 0 \text{ for } w^2 \geqslant \frac{\beta(1+\alpha^2)}{3\alpha^2(1+\beta)} \\ \frac{\frac{\beta(1+\alpha^2)}{3(1+\beta)} - \alpha^2 w^2}{2\frac{\beta(1+\alpha^2)}{3(1+\beta)} - \alpha^2 w^2} \text{ otherwise} \end{cases} \tag{4}$$

The preferred levels of the tax rates are defined over $[0, 1/2]$ and as usual in these models, it is decreasing with the level of productivity, w. Thus, independently of being a man or a woman, the level of the preferred tax rate is decreasing with w so that agents with lower productivities always prefer higher tax rate.

Moreover, for a given w, differences between $\tau^M(w)$ and $\tau^F(w)$ reflect the importance of gender gaps in longevities and productivities. To show this, let first assume that there is no difference in longevities, $\beta = 1$. In this case, the preferred tax rate of a man is always lower than the one of a woman, as he always has a higher productivity level (remember, $w_f = \alpha w$). Thus, for a given level of w, he will contribute more than a woman but obtain the same pension benefit. As a result, he always prefers a lower tax rate.

Additionally, these levels are not independent of the individual's longevity. Assume now that there is no difference in productivities between men and women ($\alpha = 1$). In this case, for a given w, the preferred tax rate level is always higher for a woman than for a man as $\beta/(1+\beta) > 1/(1+\beta)$. Since a woman has a higher expected length of life than a man, she is likely to enjoy pension benefits for a longer period than men, even though they contributed the same amount $(1-\tau)\tau w^2$. Thus, for a given productivity level w, her most preferred tax rate is higher than that of a man.

3.3 Political equilibrium

We now turn to the study of the voting equilibrium in this society with singles only. In order to determine the political equilibrium, we manipulate expressions (3) and (4) to obtain the wage rate as a function of the most preferred tax rate instead as the other way round. This yields:

$$w^M(\tau) = \sqrt{\frac{(1+\alpha^2)}{3(1+\beta)}\left(\frac{1-2\tau}{1-\tau}\right)} \quad (5)$$

$$w^F(\tau) = \frac{\sqrt{\beta}}{\alpha} w^M(\tau) \quad (6)$$

It is important that this concerns the relevant range of wages; that is the wages that yield a positive most preferred tax rate. Not surprisingly, for any given τ, $w^F(\tau) > w^M(\tau)$: the wage which supports a given level of τ is always higher for women than for men. This can be explained by the fact that women always benefit more from the pension system than men (they have both lower productivity and higher longevity) so that the level of productivity which supports a given τ is always higher for women than for men.

Remember also that since we assumed a uniform distribution of wages, $w^M(\tau)$ also gives the proportion of men with wage below w and $w^F(\tau)/\alpha$ gives the proportion of women with wage below w. Thus, having $w^F(\tau) > w^M(\tau)$ which implies $w^F(\tau)/\alpha > w^M(\tau)$, also means that the number of women in favour of a specific tax rate τ is always higher than the number of men.

We now turn to the determination of the equilibrium payroll tax rate under majority voting. The problem encountered with two characteristics is here solved by assuming that there are just two levels of longevity and that longevity and productivity were not correlated. Here, with a society comprising just men and women, an equilibrium tax rate is defined such that at least one half of the population prefers this tax rate (or a higher one) to any other lower tax rate. Since we can rank productivities, the voting equilibrium tax rate, τ^* is then such that the number of individuals with higher wage (and thus who would prefer a lower tax level) represents exactly one half of the total population:

$$w^M(\tau^*) + \frac{1}{\alpha} w^F(\tau^*) \geqslant 1$$

where a mass 1 of individuals corresponds to one half of the population. We expect that $\tau^*$is positive.[5] Replacing for $w^M(\tau^*)$ and $w^F(\tau^*)$, we obtain that

[5] In Appendix 1, we show that only an interior solution is plausible.

the equilibrium tax rate is equal to

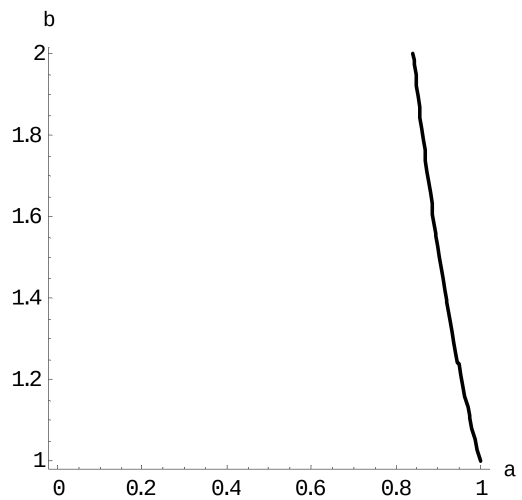
$$\tau^{s*}(\alpha,\beta)=\frac{1-\frac{1}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^2}\frac{3(1+\beta)}{(1+\alpha^2)}}{2-\frac{1}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^2}\frac{3(1+\beta)}{(1+\alpha^2)}} \tag{7}$$

where the superscript s stands for singles. As we see the voting outcome is such that it depends on both the levels of longevity gap (β) and on the productivity gap (α). To see more clearly the effects of each parameter, let first note that for $\alpha = 1$ and $\beta = 1$, the voting outcome is $\tau^{s*} = 1/5$. In appendix, we also show that

$$\partial\tau^{s*}(\alpha,\beta)/\partial\alpha < 0$$

for any given β. This is an effect of empoverishment: women become relatively poorer and there is a pressure for an increased tax rate that compensates for the decline in the tax base.

In contrast the effect on the tax rate of an increase in the longevity gap is ambiguous ($\partial\tau^{s*}(\alpha,\beta)\,\partial\beta \lesseqgtr 0$). It depends on the level of (α,β). In the following graph, we depict the curve representing all the combinations (α,β) such that $\partial\tau^{s*}(\alpha,\beta)\,\partial\beta = 0$:



To the left of (or below) that curve, $\frac{\partial\tau^{s*}(\alpha,\beta)}{\partial\beta} < 0$ and to the right, $\frac{\partial\tau^{s*}(\alpha,\beta)}{\partial\beta} > 0$. The curve is negatively sloped implying that starting from the point (1,1) for which $\tau^{s*}(\alpha,\beta) = 0.2$ one has a relation of substitution between the two parameters. To keep the derivatives equal to 0, one has to compensate an increase in longevity for women by a decrease of their wage. Let us note that in two polar cases, we have

$$\begin{aligned}\frac{\partial \tau^{s*}(\alpha,\beta)}{\partial \beta} &< 0 \text{ when } \alpha \to 1\\ \frac{\partial \tau^{s*}(\alpha,\beta)}{\partial \beta} &\to 0^{+} \text{ when } \alpha \to 0\end{aligned}$$

The first inequality is intuitive: with equal wage, the tax decreases with longevity gap as the budget pressure increases. In the other limit case where $\alpha \to 0$, gender gap in productivity is maximum and all women are equal in the extreme poverty. Regardless of β they vote for the maximum tax rate along with the few men with 0 productivity.

4 Couples and singles.

We now introduce couples. For the time being we assume that both wife and husband work and further we assume assortative mating, so that a man with productivity w always get married with a woman with productivity αw. In a following section, we deal with the problem of *one-breadwinners couples*.

4.1 Couples' preferred tax rate

We asume that spouses play cooperatively and share their resources over their life-cycle by solving the following problem:

$$\begin{aligned}&\max 2c - l_f^2/2 - l_m^2/2 + (\pi_f + \pi_m)\, u(d) \qquad (A)\\ &\text{s.to } (w_m l_m + w_f l_f)(1-\tau) + (\pi_f + \pi_m)\, p \geq 2c + (\pi_f + \pi_m)\, d\end{aligned}$$

where d represents the individual level of consumption in the second period for both members of the couple. The labour supply for each member is simply $l_m^* = w(1-\tau)$ and $l_f^* = \alpha w(1-\tau)$. Note, that under our assumptions, the labour supply of men and of women are independant of their marital condition (whether they live as a couple or whether they are single).[6] Hence, in any case, the labour supply of a woman is always lower than that of a man. This implies that her total contributions to the pension system ($\alpha w \tau$) are also lower while they receive a higher total pension benefit $\beta \pi p \geq \pi p$.

[6] A straightforward extention will consist in assuming that marital condition affects the labour supply decision.

Substituting for optimal labor supply and savings, we define the indirect utility function of a couple as

$$\begin{aligned} V^c(\tau) &= \frac{(1-\tau)^2}{2}\left(1+\alpha^2\right)w^2 - (1+\beta)\pi d^* \\ &\quad + (1+\beta)\pi p(\tau) + (1+\beta)\pi u(d^*) \end{aligned} \quad (8)$$

where d^* is the optimal consumption level in the second period and $p(\tau)$ is defined by (2). It indeed happens that the revenue constraint is the same as the one we have with singles only. In appendix 2, we find that couples with $w \geq \sqrt{1/3}$ always prefer a zero tax rate while for couples with $w < \sqrt{1/3}$, the most preferred tax rate $\tau^c(w)$ is:

$$0 < \tau^c(w) = \frac{1/3 - w^2}{2/3 - w^2} \leq 1/2$$

Note that β and α are absent from this expression so that, inside the couple, differences in longevities and in productivities between men and women do not affect the choice of a specific tax rate.[7] In a sense, the couple pools the gender differences. This is in part due to the single budget constraint and the no-liquidity-constraint assumption. In the couple, the level of the preferred tax rate only depends on the level of the husband's wage rate.

4.2 Majority voting with couples only

In an economy with only couples, the determination of the political equilibrium is simple. Since the indirect utility function $V^c(\tau)$ is convave for any $w^2 < 1/3$, we can apply the median voter theorem. Hence, the voting outcome corresponds to the preferred tax rate of the median type individual: $\tau^{*c} = \tau^c(w_m)$. Since the preferred tax rate of a couple is

$$\tau^c(w) = \begin{cases} \dfrac{1/3 - w^2}{2/3 - w^2} > 0 \text{ if } w^2 < Ew^2 \\ 0 \qquad\qquad\qquad \text{if } w^2 \geq Ew^2 \end{cases} \quad (9)$$

Under our assumption of a uniform distribution, $w^m = \bar{w} = 1/2$ so that $\tau^{*c} = \tau^c(\bar{w}) = 1/5$.

4.3 Tax rates with couples only and singles only.

In this subsection, we want to compare the values of the tax rates in these two situations, i.e. in a society where there are only single individuals and

[7] Note that if $\beta = \alpha = 1$, we have that $\tau^M(w) = \tau^F(w) = \tau^c(w)$.

where there are only couples. For this purpose, we compare $\tau^{*,c} = 1/5$ which corresponds to the voting outcome in a society where there are only couples with the voting outcome in a society where there are only singles. We know that with couples only the tax rate is constant and equal to 1/5. We also know that with singles only it varies according to the vector (α, β). In particular we know that:

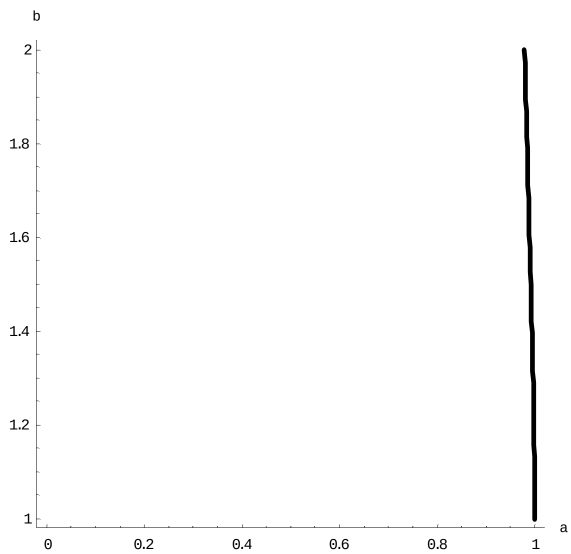
$$\tau^{s*}(1, \beta > 1) \leq \tau^{*c} = \frac{1}{5} = \tau^{s*}(1,1) \leq \tau^{s*}(\alpha < 1, 1)$$

In a society where men and women have the same productivities but different longevities, the preferred tax rate is smaller than in a society where there are only couples. On the other hand, if they all had the same longevities but different productivities, the preferred tax rate in a society with only singles would be higher than the one emerging in a society with only couples. But these are very particular cases.

For any situation with $\alpha < 1$ and $\beta > 1$, whether the tax rate in a society with singles only is greater or lower than the one obtained in a society with couples only will depend on the relative value of our two gender gaps. To this purpose, we equalize (7) to 1/5 and obtain that for any combination (α, β) such that the following equation,

$$\frac{(1+\beta)}{(1+\alpha^2)} = \frac{1}{4}\left(1 + \frac{\sqrt{\beta}}{\alpha^2}\right)^2$$

is satisfied, we have that $\tau^{*,c} = \tau^{s*}$. The following graph represents the function $\beta(\alpha)$ such that $\tau^{*c} = \tau^{s*}$,



Any combination (α, β) on the curve leads to $\tau^{*,c} = \tau^{*,s} = 1/5$. It is possible to show that below the curve, $\tau^{*c} = 1/5 > \tau^{s*}$ while above it, $\tau^{*c} = 1/5 < \tau^{s*}$.[8] This curve is downward sloping indicating that the two parameters are substitutes to maintain the tax rates equality. Using a numerical example, we can see better the two regions. The following table gives the value of τ^{s*} for $\alpha \leq 1$, $\beta \geq 1$. One sees that for $\alpha = \beta = 1$, the tax rate for singles only is the same as for couples only. The italics correspond to the area where the tax rate for singles only is lower that 0.20.

$\beta \backslash \alpha$	**1**	**0.9**	**0.8**	**0.6**	**0.5**	**0.3**	**0.1**
1	*0.20*	0.2516	0.3069	0.4086	0.4469	0.4904	0.49985
1.25	*0.1985*	0.2547	0.3125	0.4137	0.4504	0.4913	0.49987
1.5	0.1951	0.2556	0.3157	0.4171	0.4528	0.4918	0.49988
2	*0.1856*	0.2540	0.3185	0.4211	0.4558	0.4925	0.49989
5	*0.1233*	0.2289	0.3134	0.4271	0.4607	0.4937	0.499912
8	0.0732	0.2068	0.3053	0.4276	0.4617	0.4940	0.499917
10	*0.0454*	*0.1948*	0.3006	0.4274	0.4619	0.4941	0.499919

Note that this table confirms the results we had obtained in the preceding subsection, that $\partial\tau^{s*}(\alpha, \beta)/\partial\alpha < 0\,\forall\beta$, $\partial\tau^{s*}(1, \beta)\,\partial\beta < 0$ and $\partial\tau^{s*}(\alpha, \beta)\,\partial\beta \rightarrow 0$ when $\alpha \rightarrow 0$. It is also interesting to note that for $\alpha = 0.9$ and 0.8, the tax rate is first decreasing in β and then increasing.

To sum up, whether the tax rate is greater in a society made only of singles than in a society of couples only, depends on the relative values of the two gender gaps. This has implications in terms of public policy. For instance, once knowing the size of these differences, the government could decide to encourage or to discourage marriage as a way to increase or reduce the size of the pension system.

4.4 The case of a mixed society

In this section, we study the political equilibrium under a more realistic situation where the society comprises both couples and singles. The voting equilibrium process is similar to the one of Section 3.1 except that we now allow for the coexistence of couples and singles. To this purpose, we first

[8] Some points below the curve correspond to combinations where $\beta < 1$ and $\alpha = 1$. From previous computations, we know that in this case, $\tau^{*c} > \tau^{s*}$. This is the case for any point below the curve. Same reasoning can be applied above the curve: for any combination such that $\beta = 1$ and $\alpha < 1$, $\tau^{*c} < \tau^{s*}$; this will be the case for any point above the curve.

define φ as the proportion of couples in a society. In this case, the equilibrium tax rate, τ^{MI*} is such that

$$(1-\varphi)\left[w^M\left(\tau^{MI*}\right)+\frac{w^F\left(\tau^{MI*}\right)}{\alpha}\right]+2\varphi w^C\left(\tau^{MI*}\right)\geq 1 \qquad (10)$$

where $w^C(\tau)$ is the wage of a couple corresponding to a given preferred level of the tax rate and the superscript MI stands for mixed. The relation $w^C(\tau)$ is obtained from (9) and is equal to

$$w^C(\tau)=\sqrt{\frac{1}{3}\left(\frac{1-2\tau}{1-\tau}\right)}$$

Equation (10) simply says that at the voting equilibrium tax rate of a mixed society τ^{MI*}, the number of individuals with higher wage (and thus who would prefer a lower tax level) represents exactly one half of the total population. Note that if $\varphi=0$, we are back to the preceding section, in which we assumed that the society was composed of singles only. On the contrary, if $\varphi=1$, the society comprises only couples and the Condorcet winner corresponds to the tax rate preferred by the individual-couple with median productivity, $w^m=1/2$. For any intermediate case, we solve equation (10) and show in appendix E that the equilibrium tax rate is equal to

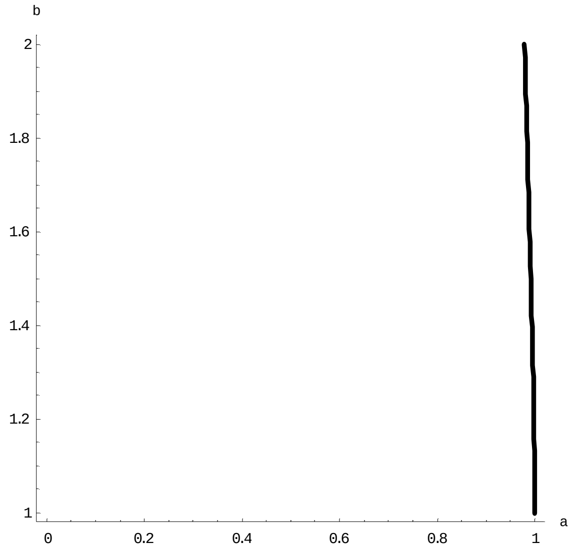
$$\tau^{MI*}(\alpha,\beta,\varphi)=\frac{1-\frac{3}{\left[(1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\right]^2}}{2-\frac{3}{\left[(1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\right]^2}} \qquad (11)$$

It is straightforward to show that at $\alpha=\beta=1$, $\tau^{MI*}(1,1,\varphi)=1/5$. In this case, when there are no gender differences, the tax rate corresponds to the one preferred by the individual with median productivity and does not depend on φ.[9] Thus, only the distribution of productivities matters and the political equilibrium is independent on φ.

In the more likely case where there exist gender differences, the structure of the society (i.e. the fraction φ) matters for the determination the political equilibrium and the size of the pension system. First, we compare $\tau^{*c}=1/5$ and $\tau^{s*}(\alpha,\beta)$ with (11) and show in the appendix, that there exists a function $\beta(\alpha)$ which represents all the combinations (α,β) for which

[9] Under our assumption of a uniform distribution of wages, it simply equal to $\left(1/3-\bar{w}^2\right)/\left(2/3-\bar{w}^2\right)$ with $\bar{w}=w^m=1/2$ (equivalent to $\tau^{*c}=\tau^{*s}(1,1)$).

$\tau^{MI*}(\alpha,\beta,\varphi) = \tau^{*c} = \tau^{s*}(\alpha,\beta)$:[10] Not surprisingly this curve is negatively slope.



To the left of this curve we have the following inequalities,

$$\tau^{*c} \leq \tau^{MI*}(\alpha,\beta,\varphi) \leq \tau^{s*}(\alpha,\beta)$$

This would be the case if, for example, there were no difference in longevities but in productivities ($\alpha \leq 1$ and $\beta = 1$). In the appendix, we also show that in this region, when the proportion φ of couples increases, the voting equilibrium tax rate decreases, $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\varphi \leq 0$. On the contrary, to the right of the curve, we have $\tau^{s*}(\alpha,\beta) \leq \tau^{MI*}(\alpha,\beta,\varphi) \leq \tau^{*c} = 1/5$ so that an increase in the number of couples would increase the equilibrium tax rate (i.e. $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\varphi \leq 0$). This would be the case in a society with no gender difference in productivities but in longevities, i.e. $\alpha = 1$ and $\beta \geq 1$.

Finally, like in the preceding section, we make comparative statics with respect to α, β and φ. We show in the appendix, that $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\alpha < 0$ and $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\beta \gtrless 0$ with $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\beta < 0$ when $\alpha \to 1$ and $\partial\tau^{MI*}(\alpha,\beta,\varphi)/\partial\beta > 0$ when $\alpha \to 0$. This is similar to the results we had obtained in section 3.3, which seems reasonable as the only difference here in this section, is to allow for the coexistence of couples and of singles, knowing that couples do not care about gender differences when voting for the level of the tax rate. Thus, when the proportion of couple φ is fixed, if parameters α or β vary, this will modify only the preference for the tax rate levels of

[10]Note that this curve is exactly the same than the one we had obtained in Figure 1.

single men and of single women. Hence, the voting outcome will only reflect the change in preferences of these categories (which are equivalent to what we had described in the previous section).

5 Extension: allowing for 1vs 2 breadwinners (to be completed)

5.1 The problem of a 1 breadwinner couple

The problem of *a couple with only 1 breadwinner* is slightly different from the case with two breadwinners (problem A). To keep thinks simple we assume that some couples are made of one-breadwinners and that the breadwinner is the husband. We further assume that in such a couple the wife is entitled to a certain benefit: a fraction $\gamma \in [0,1]$ of the per period pension benefit for π_f periods.[11] We can write the problem of this couple as:

$$\begin{aligned} &\max 2c - l_m^2/2 + (\pi_f + \pi_m)\, u(d) \\ &\text{s.to } w_m l_m (1-\tau) + (\gamma\pi_f + \pi_m)\, p \geq 2c + (\pi_f + \pi_m)\, d \end{aligned} \tag{B}$$

As assumed, only the man supplies labour and $l_m^* = w(1-\tau)$. Replacing for π_m and π_f, the indirect utility function is thus:

$$V^{c,1bw}(\tau,p) = \frac{w^2(1-\tau)^2}{2} + (1+\gamma\beta)\,\pi p - (\beta+1)\,\pi d^* + (\beta+1)\,\pi u(d^*) \tag{12}$$

5.2 General pension benefit formula

With this new component, the revenue constrqint and the ensueing pension formula have to be modified. We assume that there exists an equal fraction, $(1-\varphi)$ of single males and single females; there is a fraction φ of couples, so that a number 2φ of individuals live in couple. Among these couples, we assume that a number b of couples are composed of two breadwinners, while a number $(1-b)$ are constituted of only one breadwinner

In order to define the pension equation, let first look at the total contributions level:

$$TC = w_m l_m \tau + w_f l_f (1-\varphi+\varphi b)$$

In this model, whatever their marital situation, men supply labour, but only single women and women belonging to a two-breadwinner couple supply

[11] This is realistic as many pension schemes provide higher replacement rates to a one earner couple than to a single individual.

labour and thus pay contributions. Replacing for optimal labour supplies, we have

$$TC = \frac{1}{3}(1-\tau)\tau\left[1+\alpha^2(1-\varphi+\varphi b)\right]$$

On the benefit side, we use the same reasoning,

$$TB = \pi_m p + p\pi_f(1-\varphi+\varphi b+\gamma\varphi(1-b))$$

where $\gamma \in [0,1]$ represents a fraction of the full pension benefit, which we assume to be received by the spouce of a one-breadwinner couple. Here, every contributors (i.e all men, single women and women from a two-breadwinner couple) receive a full pension benefit, p as before but we also allow women of one breadwinner couple to receive, γp. Substituting for $\pi_f = \beta\pi$, we get

$$TB = p\pi\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]$$

From budget balance, we have

$$p(\alpha,\beta,\tau,\varphi,b) = \frac{(1-\tau)\tau\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3\pi\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]} \quad (13)$$

Note that if we assume a society of only singles ($\varphi = 0$) or a society of only couples with two breadwinners ($\varphi = 1$ and $b = 1$), p is equal to (2)and we are back to our previous sections.

5.3 Political equilibrium

We now have four different categories of individuals: single men, single women, couples with one breadwinner and couples with two breadwinners. In the previous sections, we already studied the cases of a society with only singles ($\varphi = 0$), with only two-breadwinner couples ($\varphi = 1$, $b = 1$) and with singles and two-breadwinner couples ($\varphi \in]0,1[$, $b = 1$). In this section, we are going to study successively cases where there are

- only one-breadwinner couples, $\varphi = 1$ and $b = 0$
- singles and one-breadwinner couples, $\varphi \in]0,1[$ and $b = 0$
- singles, one-breadwinner and two-breadwinner couples, $\varphi \in]0,1[$ and $b \in [0,1]$

5.3.1 A society with only one breadwinner couples

Let first rewrite the pension benefit with $\varphi = 1$ and $b = 0$,

$$p(\alpha, \beta, \tau, 1, 0) = \frac{(1-\tau)\tau}{3\pi[1+\beta\gamma]}$$

Replacing it in (12), we get

$$V^{c,1bw}(\tau, p) = \frac{w^2(1-\tau)^2}{2} + \frac{(1-\tau)\tau}{3} - (\beta+1)\pi d^* + (\beta+1)\pi u(d^*)$$

in this case, the preferred tax rate of a one-breadwinner couple corresponds to

$$\arg\max_{\tau\in[0,1]} V^{c,1bw}(\tau)$$

and it is possible to show that

$$\tau^{c,1bw}(w) = \frac{1/3 - w^2}{2/3 - w^2}$$

In a society where there are only couples, the preferred tax rate level is exactly the same as in a society with only two-breadwinner couples and then independant of gender differences. This result is simply due to the fact that *every* individuals will contribute $w^2(1-\tau)\tau$ and receive $(1+\beta\gamma)p$. Since every couples are in the same situation, it does not create additional differences between couples and it is logical to find the same preferred tax rate level. In this case, the voting outcome is $\tau^*(\alpha, \beta, 1, 0, \gamma) = 1/5$.

5.3.2 The political equilibrium in a society of singles and one-breadwinner couples

As before, we rewrite the pension benefit so as to take into account the composition of the society; here $\varphi \in]0, 1[$ and $b = 0$

$$p(\alpha, \beta, \tau, \varphi, 0) = \frac{(1-\tau)\tau[1+\alpha^2(1-\varphi)]}{3\pi[1+\beta(1-\varphi+\gamma\varphi)]}$$

In this case, we first have to compute the preferred tax rate for single individuals and for one-breadwinner couples, taking into account thechange in the pension benefit formula.

For *singles*, where ** accounts for an economy with singles and one-breadwinner couples. Hence, in the appendix, we show that the preferred tax rates for single men and women have the following expressions

$$\tau^{M**}(w) = \begin{cases} 0 \text{ for } w^2 \geq \frac{\left[1+\alpha^2(1-\varphi)\right]}{3[1+\beta(1-\varphi+\gamma\varphi)]} \\ \frac{\frac{\left[1+\alpha^2(1-\varphi)\right]}{3[1+\beta(1-\varphi+\gamma\varphi)]}-w^2}{2\frac{\left[1+\alpha^2(1-\varphi)\right]}{3[1+\beta(1-\varphi+\gamma\varphi)]}-w^2} \text{ otherwise} \end{cases}$$

$$\tau^{F**}(w) = \begin{cases} 0 \text{ for } w^2 \geq \frac{\left[1+\alpha^2(1-\varphi)\right]\beta}{3\alpha^2[1+\beta(1-\varphi+\gamma\varphi)]} \\ \frac{\frac{\left[1+\alpha^2(1-\varphi)\right]\beta}{3[1+\beta(1-\varphi+\gamma\varphi)]}-\alpha^2 w^2}{2\frac{\left[1+\alpha^2(1-\varphi)\right]\beta}{3[1+\beta(1-\varphi+\gamma\varphi)]}-\alpha^2 w^2} \text{ otherwise} \end{cases}$$

We find that $\tau^{M**}(w) \leq \tau^{M}(w)$ (which corresponds to expressions 3) when gender differences are both not very important ($\alpha \to 1$ and $\beta \to 1$) and that $\tau^{M**}(w) \geq \tau^{M}(w)$ when one or the other gender difference is very important (i.e. $\alpha = 1$ and β very high or $\beta = 1$ and $\alpha \to 0$). This simply means in a society with only one earner, the preferred tax rate of men is likely to be higher than in a society with only singles or in a mixed society with two breadwinners, as long as gender differences are high.

[to be done: with women]

We also have to find the preferred tax rate for a couple. A one-breadwinner couple will maximise

$$\begin{aligned} V^{c,1bw}(\tau) &= \frac{w^2(1-\tau)^2}{2} + (1+\gamma\beta)\frac{(1-\tau)\tau\left[1+\alpha^2(1-\varphi)\right]}{3\left[1+\beta(1-\varphi+\gamma\varphi)\right]} \\ &\quad - (\beta+1)\pi d^* + (\beta+1)\pi u(d^*) \end{aligned}$$

where we replaced for $p(\alpha, \beta, \tau, \varphi, 0)$. The preferred tax rate is

$$\tau^{c**}(w) = \frac{\frac{(1+\gamma\beta)\left[1+\alpha^2(1-\varphi)\right]}{3[1+\beta(1-\varphi+\gamma\varphi)]} - w^2}{2\frac{(1+\gamma\beta)[1+\alpha^2(1-\varphi)]}{3[1+\beta(1-\varphi+\gamma\varphi)]} - w^2}$$

As compared to the previous section with two-breadwinners couples, the preferred tax rate of couples now depends on gender differences. As before, the political equilibrium is defined by

$$(1-\varphi)\left[w^{M**}(\tau) + \frac{w^{F**}(\tau)}{\alpha}\right] + 2\varphi w^{C**}(\tau) \geq 1$$

where $w^{M**}(\tau)$, $w^{F**}(\tau)$ and $w^{C**}(\tau)$ are obtained from the inversion of the above preferred tax rates. In appendix, we show that the equilibrium tax rate is equal to

$$\tau^*(\alpha,\beta,\varphi,0,\gamma)=\frac{1-\frac{3[1+\beta(1-\varphi+\gamma\varphi)]}{[1+\alpha^2(1-\varphi)]\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]^2}}{2-\frac{3[1+\beta(1-\varphi+\gamma\varphi)]}{[1+\alpha^2(1-\varphi)]\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]^2}}$$

Comparing this expression with the tax rate we obtained in a mixed society, $\tau^{M*}(\alpha,\beta,\varphi)$, we find that if there is no difference in gender, we prove in appendix that $\tau^{**}(1,1,\varphi,\gamma)$ is always ... than $\tau^{M*}(1,1,\varphi)=1/5$ for any $\gamma>0$.

[to be continued]

5.3.3 The political equilibrium in a society of singles, one-earner and two-earner couples

In this section, we assume no restriction for the composition of the society, so that $\varphi\in\,]0,1[$ and $b\in[0,1]$ and the pension benefit has the expression (13). As before, we determine the preferred tax rates for single individuals, one-earner couples and two earner couples. We obtain the following tax rates,As before, inverting these tax rates, we compute the equilibrium tax rate, which satisfies,

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Appendix

A Interior solution

to be completed

B Preferred tax rates

We solve the following program of single individuals:

$$\max_{\tau\in[0,1]} V^i(\tau) = \frac{(1-\tau)^2 w_i^2}{2} - s^* + \pi_i u\left(\frac{s}{\pi_i} + \tau\frac{(1-\tau)\,Ew^2}{\bar{\pi}}\right)$$

Deriving $V^i(\tau)$ with respect to τ, we obtain

$$\frac{\partial V^i(\tau,p)}{\partial\tau} = -(1-\tau)\,w_i^2 + \pi_i u'(d_i^*)\,\frac{Ew^2}{\bar{\pi}}(1-2\tau)$$

with $u'(d_i^*) = 1$. Taking this epression at $\tau = 0$, we find that any individual with $w_i^2 \geq Ew^2\pi_i/\bar{\pi}$ always prefers a zero tax rate. For those with $w_i^2 < Ew^2\pi_i/\bar{\pi}$, the solution is interior and the preferred tax rate is equal to (1).

C Preferred tax rate of a couple

So as to find the preferred tax rate level of a couple, we derive their indirect utility function with respect to τ,

$$\frac{\partial V^c(\tau)}{\partial \tau} = \left[\frac{(1-2\tau)}{3} - (1-\tau) w^2\right] \left(1+\alpha^2\right)$$

Taking this expression at $\tau = 0$, we find that couples with $w^2 \geq 1/3$ always prefer a zero tax rate while the solution is interior for individuals with $w^2 < 1/3$ and has the following expression

$$0 < \tau^c(w) = \frac{1/3 - w^2}{2/3 - w^2} \leq 1/2$$

D Political equilibrium in a society in which all individuals are single

Replacing for the expressions of $w^M(\tau)$ and $w^F(\tau)$ in $w^M(\tau) + w^F(\tau)/\alpha = 1$, we obtain

$$\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)\sqrt{\frac{(1+\alpha^2)}{3(1+\beta)}\left(\frac{1-2\tau}{1-\tau}\right)} = 1$$

$$\Rightarrow \quad 1-2\tau = \frac{1}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^2}\frac{3(1+\beta)}{(1+\alpha^2)}(1-\tau)$$

which yields τ^{s*}. Let us then define a function $A(\alpha, \beta)$ such that

$$\begin{aligned} A(\alpha,\beta) &= \frac{1}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^2}\frac{3(1+\beta)}{(1+\alpha^2)} \\ \text{and } \tau^{s*}(\alpha,\beta) &= \frac{1-A(\alpha,\beta)}{2-A(\alpha,\beta)} \end{aligned}$$

Thus

$$\begin{aligned} \frac{\partial \tau^{s*}(\alpha,\beta)}{\partial \alpha} &= \frac{-\partial A(\alpha,\beta)/\partial \alpha}{[2-A(\alpha,\beta)]^2} \\ \text{and } \frac{\partial \tau^{s*}(\alpha,\beta)}{\partial \beta} &= \frac{-\partial A(\alpha,\beta)/\partial \beta}{[2-A(\alpha,\beta)]^2} \end{aligned}$$

where

$$\frac{\partial A(\alpha,\beta)}{\partial\alpha} = \frac{3(1+\beta)}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^3 (1+\alpha^2)^2}\frac{2}{\alpha^3}\left[\left(2+\alpha^2\right)\sqrt{\beta}-\alpha^4\right] > 0$$

$$\text{and}\frac{\partial A(\alpha,\beta)}{\partial\beta} = \frac{3}{\left(\frac{1}{\alpha^2}+\sqrt{\beta}\right)^3 (1+\alpha^2)}\left[1-\frac{1}{\alpha^2\sqrt{\beta}}\right] \lesseqgtr 0$$

In the latter case,if $\alpha = 1$, $\partial A(1,\beta)/\partial\beta > 0$ whereas if $\alpha \to 0$, after some rearrangements, we obtain that $\partial A(\alpha,\beta)/\partial\beta \to 0$. Thus, $\partial\tau^{s*}(\alpha,\beta)/\partial\alpha < 0$ and $\partial\tau^{s*}(\alpha,\beta)\,\partial\beta \lesseqgtr 0$ with $\partial\tau^{s*}(1,\beta)\,\partial\beta < 0$ and $\partial\tau^{s*}(\alpha,\beta)\,\partial\beta \to 0$ when $\alpha \to 0$.

E Political equilibrium in a mixed society

Replacing for this expression and for $w^M(\tau)$, $w^F(\tau)$ and $w^c(\tau)$in

$$(1-\varphi)\left[w^M\left(\tau^{M*}\right)+\frac{w^F\left(\tau^{M*}\right)}{\alpha}\right]+2\varphi w^C\left(\tau^{M*}\right)=1$$

we obtain

$$(1-\varphi)\left[\sqrt{\frac{(1+\alpha^2)}{3(1+\beta)}\left(\frac{1-2\tau}{1-\tau}\right)}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)\right]+2\varphi\sqrt{\frac{1}{3}\left(\frac{1-2\tau}{1-\tau}\right)}=1$$

after some rearrangements, we obtain (11).

- Comparison of $\tau^{M*}(\alpha,\beta,\varphi)$ with τ^{*c}. We have $\tau^{M*}(\alpha,\beta,\varphi) \geq \tau^{*c}$ iff

$$\frac{1-\frac{3}{\left[(1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\right]^2}}{2-\frac{3}{\left[(1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\right]^2}} \geq \frac{1}{5}$$

$$\Leftrightarrow\ 2-\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right) \leq 0$$

after some rearrangements.

- Comparison of $\tau^{M*}(\alpha,\beta,\varphi)$ with $\tau^{s*}(\alpha,\beta)$. We have $\tau^{s*}(\alpha,\beta) \geq \tau^{M*}(\alpha,\beta,\varphi)$ if

$$\frac{1}{\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)^2}\frac{3(1+\beta)}{(1+\alpha^2)} > \frac{3}{\left[(1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\right]^2}$$
$$\Leftrightarrow \left[2-\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)\right]\varphi \leq 0$$

Thus, for any values of (α,β) such that

$$2-\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right) \leq 0 \text{ (resp. } \geq \text{)}$$

we obtain that $\tau^{*c} \leq \tau^{M*}(\alpha,\beta,\varphi) \leq \tau^{s*}(\alpha,\beta)$ (resp. $\geq$). In figure (...), we represented the function $\beta(\alpha)$ such that there is perfect equality. Note that if $\alpha = 1$, the above expression is positive for any value of β so that $\tau^{s*}(1,\beta) \leq \tau^{M*}(1,\beta,\varphi) \leq 1/5$. On the contrary, if $\beta = 1$, this expression is negative for any value of $\alpha \in [0,1]$, so that we obtain the opposite relation and $\tau^{s*}(\alpha,1) \geq \tau^{M*}(\alpha,1,\varphi) \geq 1/5$.

- We also show how (11) varies with parameters α, β and φ. First we define a function $B(\alpha,\beta,\varphi)$ as

$$B(\alpha,\beta,\varphi) = (1-\varphi)\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi$$

so that

$$\frac{\partial\tau^{M*}(\alpha,\beta,\varphi)}{\partial\alpha} = \frac{6}{B(\alpha,\beta,\varphi)^3}\frac{\partial B(\alpha,\beta,\varphi)/\partial\alpha}{\left[2-3/B(\alpha,\beta,\varphi)^2\right]^2}$$
$$\frac{\partial\tau^{M*}(\alpha,\beta,\varphi)}{\partial\beta} = \frac{6}{B(\alpha,\beta,\varphi)^3}\frac{\partial B(\alpha,\beta,\varphi)/\partial\beta}{\left[2-3/B(\alpha,\beta,\varphi)^2\right]^2}$$
$$\frac{\partial\tau^{M*}(\alpha,\beta,\varphi)}{\partial\varphi} = \frac{6}{B(\alpha,\beta,\varphi)^3}\frac{\partial B(\alpha,\beta,\varphi)/\partial\varphi}{\left[2-3/B(\alpha,\beta,\varphi)^2\right]^2}$$

where

$$\frac{\partial B(\alpha,\beta,\varphi)}{\partial\alpha} = \frac{(1-\varphi)}{\sqrt{(1+\alpha^2)(1+\beta)}}\left(\alpha-\frac{\sqrt{\beta}}{\alpha}\left(1+\frac{2}{\alpha^2}\right)\right)<0$$
$$\frac{\partial B(\alpha,\beta,\varphi)}{\partial\beta} = \frac{(1-\varphi)}{2}\frac{(1+\alpha^2)^{1/2}}{(1+\beta)^{3/2}}\left(\frac{1}{\alpha^2\sqrt{\beta}}-1\right)\gtreqless 0$$
$$\frac{\partial B(\alpha,\beta,\varphi)}{\partial\varphi} = 2-\sqrt{\frac{(1+\alpha^2)}{(1+\beta)}}\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)\gtreqless 0$$

so that $\partial\tau^{M*}(\alpha,\beta,\varphi)/\partial\alpha<0$, $\partial\tau^{M*}(\alpha,\beta,\varphi)/\partial\beta\gtreqless 0$ and $\partial\tau^{M*}(\alpha,\beta,\varphi)/\partial\varphi\gtreqless 0$. We also obtain that $\partial\tau^{M*}(\alpha,\beta,\varphi)/\partial\beta<0$ when $\alpha\to 1$ and $\partial\tau^{M*}(\alpha,\beta,\varphi)/\partial\beta>0$ when $\alpha\to 0$. Note that $\partial B(1,\beta,\varphi)/\partial\varphi> 0\forall\beta$ and $\partial B(\alpha,1,\varphi)/\partial\varphi<0$, so that $\partial\tau^{M*}(1,\beta,\varphi)/\partial\varphi>0$ and $\partial\tau^{M*}(\alpha,1,\varphi)/\partial\varphi<0$.

F political equilibrium in a society of singles and one-breadwinner couples

F.1 preferred tax rates

- For singles, the problem is to maximise

$$V^i(\tau)=\frac{(1-\tau)^2 w_i^2}{2}-s_i^*+\pi_i u\left(\frac{s_i^*}{\pi_i}+\frac{(1-\tau)\tau\left[1+\alpha^2(1-\varphi)\right]}{3\pi\left[1+\beta(1-\varphi+\gamma\varphi)\right]}\right)$$

where we replaced for the expression of $p(\alpha,\beta,\tau,\varphi,0)$. We find that the preferred tax rate is then equal to

$$\tau^{**}(w_i,\pi_i)=\frac{\frac{\left[1+\alpha^2(1-\varphi)\right]\pi_i}{3\pi[1+\beta(1-\varphi+\gamma\varphi)]}-w_i^2}{2\frac{[1+\alpha^2(1-\varphi)]\pi_i}{3\pi[1+\beta(1-\varphi+\gamma\varphi)]}-w_i^2}$$

Replacing for $\pi_f=\beta\pi$, $\pi_m=\pi$, $w_f=\alpha w$ and $w_m=w$, we obtain $\tau^{M**}(w)$ and $\tau^{F**}(w)$.

- The following equation defines the voting outcome

$$(1-\varphi)\left[w^{M**}(\tau)+\frac{w^{F**}(\tau)}{\alpha}\right]+2\varphi w^{C**}(\tau)\geq 1$$

with

$$\begin{aligned} w^{M**}(\tau) &= \sqrt{\frac{\left[1+\alpha^2(1-\varphi)\right]}{3\left[1+\beta(1-\varphi+\gamma\varphi)\right]}\left(\frac{1-2\tau}{1-\tau}\right)} \\ w^{F**}(\tau) &= \frac{\sqrt{\beta}}{\alpha}w^{M**}(\tau) \\ w^{c**}(\tau) &= \sqrt{(\beta+\gamma)}w^{M**}(\tau) \end{aligned}$$

Replacing for these expressions, the above inequality is

$$\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]\sqrt{\frac{\left[1+\alpha^2(1-\varphi)\right]}{3\left[1+\beta(1-\varphi+\gamma\varphi)\right]}}\sqrt{\left(\frac{1-2\tau}{1-\tau}\right)}=1$$

$$\left(\frac{1-2\tau}{1-\tau}\right)=\frac{1}{\frac{[1+\alpha^2(1-\varphi)]}{3[1+\beta(1-\varphi+\gamma\varphi)]}\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]^2}$$

$$\tau^{**}=\frac{1-\frac{3[1+\beta(1-\varphi+\gamma\varphi)]}{[1+\alpha^2(1-\varphi)]\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]^2}}{2-\frac{3[1+\beta(1-\varphi+\gamma\varphi)]}{[1+\alpha^2(1-\varphi)]\left[(1-\varphi)\left(1+\frac{\sqrt{\beta}}{\alpha^2}\right)+2\varphi\sqrt{(\beta+\gamma)}\right]^2}}$$

- *Comparison of* $\tau^{**}(1,1,\varphi,\gamma)$ *with* $\tau^{M*}(1,1,\varphi)$*:*

G Political equilibrium with no restriction on the composition of the society

G.1 Preferred tax rates

- Single individuals maximise

$$V^i(\tau)=\frac{(1-\tau)^2w_i^2}{2}-s_i^*+\pi_i u\left(\frac{s_i^*}{\pi_i}+\frac{(1-\tau)\tau\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3\pi\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]}\right)$$

which yields,

$$\tau^{***}(w_i,\pi_i)=\begin{cases} 0 \text{ for } w_i^2>\frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]\pi_i}{3\pi[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]} \\ \frac{\frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]\pi_i}{3\pi[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w_i^2}{\frac{2\left[1+\alpha^2(1-\varphi+\varphi b)\right]\pi_i}{3\pi[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w_i^2} \text{ otherwise} \end{cases}$$

so that

$$\tau^{M***}(w) = \begin{cases} 0 \text{ for } w^2 \geq \frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]} \\ \frac{\frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w^2}{\frac{2\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w^2} \text{ otherwise} \end{cases}$$

$$\tau^{F***}(w) = \begin{cases} 0 \text{ for } w^2 \geq \frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]\beta}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]} \\ \frac{\frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]\beta}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-\alpha^2 w^2}{\frac{2\left[1+\alpha^2(1-\varphi+\varphi b)\right]\beta}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-\alpha^2 w^2} \text{ otherwise} \end{cases}$$

- Replacing for the expression of the pension benefit (13) into (12), one-earner couples maximise

$$\begin{aligned} V^{c,1bw}(\tau) &= \frac{w^2(1-\tau)^2}{2} + (1+\gamma\beta)\,\pi\frac{(1-\tau)\,\tau\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3\pi\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]} \\ &\quad -2\pi d^* - (\beta-1)\,\pi d_s^* + 2\pi u(d^*) + (\beta-1)\,\pi u(d_s^*) \end{aligned}$$

which yields

$$\tau^{1bw***}(w) = \begin{cases} 0 \text{ for } w^2 > \frac{(1+\gamma\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]} \\ \frac{\frac{(1+\gamma\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w^2}{\frac{2(1+\gamma\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-w^2} \text{ otherwise} \end{cases}$$

- Two-earner couples maximise (8) where we replaced $p(\tau)$ by (13):

$$\begin{aligned} V^{c,2bw}(\tau) &= \frac{(1-\tau)^2}{2}\left(1+\alpha^2\right)w^2 - 2d^*\pi_m - (\pi_f-\pi_m)\,d_s^* \\ &\quad + \frac{(1+\beta)(1-\tau)\,\tau\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]} + 2\pi_m u(d^*) + (\pi_f-\pi_m)\,u(d_s^*) \end{aligned}$$

Differentiating this expression with respect to τ, we get that the preferred tax rate of a couple is

$$\tau^{C,2bw***}(w) = \begin{cases} 0 \text{ for } w^2 > \frac{(1+\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3(1+\alpha^2)[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]} \\ \frac{\frac{(1+\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-\left(1+\alpha^2\right)w^2}{\frac{2(1+\beta)\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))]}-(1+\alpha^2)w^2} \text{ otherwise} \end{cases}$$

- the equilibrium tax rate is defined by

$$(1-\varphi)\left[w^{M***}(\tau)+\frac{w^{F***}(\tau)}{\alpha}\right]+2\varphi\left[(1-b)\,w^{1bw***}(\tau)+bw^{2bw***}(\tau)\right]\geq 1$$

with

$$\begin{aligned}
w^{M***}(\tau) &= \sqrt{\frac{\left[1+\alpha^2(1-\varphi+\varphi b)\right]}{3\left[1+\beta(1-\varphi+\varphi b+\gamma\varphi(1-b))\right]}\left(\frac{1-2\tau}{1-\tau}\right)} \\
w^{F***}(\tau) &= \frac{\sqrt{\beta}}{\alpha}w^{M***}(\tau) \\
w^{1bw***}(\tau) &= \sqrt{(1+\gamma\beta)}w^{M***}(\tau) \\
w^{2bw***}(\tau) &= \sqrt{\frac{1+\beta}{1+\alpha^2}}w^{M***}(\tau)
\end{aligned}$$