# The Condorcet Jury Theorem and Committee Design

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# Overview

Voting does two things: aggregates preferences and aggregates information

preference aggregation problematic only when preferences heterogeneous or there are at least three alternatives

information aggregation problematic even with homogeneous preferences and two alternatives

Three lectures to come survey the as yet far less extensive literature on information aggregation in voting:

**Lecture 1:** Information aggregation and voting in small committees; committee design

**Lecture 2:** Information aggregation and voting in large elections

**Lecture 3:** Back to small committees but with talking before voting.

# I The Basic Model

Individuals: $N = \{1, \dots, n\}$, $n$ odd

Alternatives: $X = \{A, B\}$

States of the world: $\Omega = \{A, B\}$

Preferences: $\forall i \in N$,

$u_i(x, \omega) =$

| | $A$ | $B$ |
|---|---|---|
| $A$ | $0$ | $-(1-t)$ |
| $B$ | $-t$ | $0$ |

where $t \in (0, 1)$.

Common prior belief: $\Pr[\omega = A] = \pi \in (0, 1)$

Each $i \in N$ privately observes a conditionally independent signal $s_i \in \{a, b\}$: $\forall i \in N$,

$$\Pr[s_i = z | \omega = Z] = p_Z \in (\frac{1}{2}, 1), \;\; (z, Z) \in \{(a, A), (b, B)\}.$$

$i$'s voting strategy, $v_i : \{a, b\} \to [0, 1]$; $v_i(s_i) \equiv \Pr[i \text{ votes } B | s_i]$.

Assume no abstention and focus throughout on Bayesian equilibria in weakly undominated strategies.

**Definition**

Voting is *informative* if and only if $v_i(a) = 1 - v_i(b) = 0$.

Let $x^*$ be the ex post best alternative and $\mathbf{v} = (v_1, \ldots, v_n)$. Then,

$\Pr[i \text{ votes } x^* | v_i \text{ informative}] \equiv \hat{p} = \pi p_A + (1 - \pi) p_B$;

$\Pr[i \& j \text{ vote } x^* | \mathbf{v} \text{ informative}] \equiv r = \pi p_A^2 + (1 - \pi) p_B^2$.

$\implies$ independence iff $p_A = p_B = p$.

# II The Condorcet Jury Theorem

### Theorem (Condorcet Jury Theorem)

*Let $p_A = p_B = p > 1/2$ and assume* $\mathbf{v}$ *informative. The probability that a majority votes for the ex post best alternative is strictly higher than the probability any individual selects that alternative. Furthermore, as $n \to \infty$, the majority probability goes to one.*

Let $p_{iZ} = \Pr[s_i = z | \omega = Z]$ and define:

$$\bar{p} \equiv \frac{1}{n} \sum_i \left[\pi p_{iA} + (1-\pi) p_{iB}\right]; \ \bar{r} \equiv \frac{1}{n} \sum_i \left[\pi p_{iA}^2 + (1-\pi) p_{iB}^2\right].$$

### Theorem (Ladha 1992)

*Assume $\bar{p} > 1/2$ and $\mathbf{v}$ informative. Then there exists $r(\bar{p}, n)$ such that the CJT goes through for all $\bar{r} < r(\bar{p}, n)$.*

Further extensions include Nitzan & Paroush (1985); Miller (1986); Young (1988); Grofman & Feld (1988); Berg (1993); Ben-Yashar & Paroush (2000); Berend & Sapir (2007).

# III Rational Voting

### Definition

Voting is *sincere* if $v_i(s_i) > 0$ implies
$E[u_i(B, \omega)|s_i] \geq E[u_i(A, \omega)|s_i]$.

Given $\pi$, let $k^*$ be the maximum number of "$b$"-signals that any individual $i$ could observe and still prefer $A$ to $B$.

Assume decisions made under a *q-rule*, $q \leq n$, whereby $B$ is chosen iff if there are *at least* $q$ votes for $B$.

**Examples** $q = (n + 1)/2$ is simple majority rule; $q = n$ is unanimity rule.

### Theorem (Austen-Smith & Banks 1996)

*Let $p_A = p_B = p > 1/2$. Sincere voting is informative and constitutes Nash equilibrium behaviour in the underlying voting game induced by $q$ iff $q - 1 = k^* = (n-1)/2$.*

**Intuition** strategically rational agents condition vote on being pivotal, and the event "$i$ is pivotal" conveys information.

For later reference, call $q^S = k^* + 1$ the *statistical rule*.

**Proof** [Sketch] $p_A = p_B$ implies sincere voting is informative iff

$$\frac{p}{1-p} > \frac{\pi}{1-\pi}\left(\frac{t}{1-t}\right) > \frac{1-p}{p}.$$

The integer $k^*$ is defined implicitly by,

$$\left[\frac{p}{1-p}\right]^{2(k^*+1)-n} > \frac{\pi}{1-\pi}\left(\frac{t}{1-t}\right) > \left[\frac{1-p}{p}\right]^{n-2k^*}.$$

These inequalities imply sincere voting is informative iff $k^* = (n-1)/2$.

To see that informative voting is equilibrium behaviour iff $q = k^* + 1$, consider the following example:

Assume $N = \{1, 2, 3\}$ and 1, 2 vote informatively.

(1) If $k^* = 2$ and *majority rule*, 3 is pivotal (and so her vote matters) only when 1 and 2 vote against each other.

That is, given $(v_1, v_2)$ informative, 3 pivotal iff exactly one "$b$"-signal is observed by 1 and 2.

Therefore, 3's best response is $v_3^*(s_3) = 0$ for all $s_3 \in \{a, b\}$.

(2) If $k^* = 1$, then 3's best response under majority rule (conditional on 3 pivotal) is informative and sincere.

(3) If $k^* = 1$ and *unanimity rule*. Given $(v_1, v_2)$ informative, 3 is pivotal iff *both* 1 and 2 observe the "$b$"-signal; then $v_3^*(s_3) = 1$ for all $s_3 \in \{a, b\}$. $\square$

## Definition

A voting profile $\mathbf{v} = (v_1, \ldots, v_n)$ is *symmetric* iff $v_i = v_j$, all $i, j$.

## Theorem (McLennan 1998)

*Fix a q-rule and any $p_A > 1/2$, $p_B > 1/2$. If informative voting exhibits the CJT properties, then there exists a symmetric mixed strategy voting equilibrium that likewise exhibits the CJT properties.*

**Intuition** Consider the problem, $\max_{\mathbf{v}} E[u_i(f_q(\mathbf{v}), \omega)|\mathbf{s}]$, where $\mathbf{s} = (s_1, \ldots, s_n)$ is the realized signal profile and $f_q$ maps realizations of the (possibly mixed) strategies $\mathbf{v} = (v_1(s_1), \ldots, v_n(s_n))$ into outcomes under the given $q$-rule.

By common values, the solution $\mathbf{v}$ is a Bayesian equilibrium to the voting game and as such weakly dominates informative voting.

# IV Unanimity Rule

Convenient (but unnecessary) to assume $\pi = 1/2$ and $p_A = p_B = p$

### Definition

A symmetric mixed strategy $v$ is *responsive* if, with strictly positive probability, the likelihood that any individual votes $B$ when the true state is $B$ differs from that when the true state is $A$.

Let $z_n(x, \omega) = \Pr[x \text{ chosen} |\ \omega, n]$

## Theorem (Feddersen & Pesendorfer 1998)

*Suppose the conditional probability that* $\omega = B$ *given* $(n-1)$ *"b"-signals is greater than* $t$.
*(1) When* $t > 1 - p$ *there exists a unique responsive symmetric equilibrium* $\mathbf{v}^*$ *for the unanimity rule. Moreover*

$$\begin{aligned} v^*(b) &= \lim_{n\to\infty} v^*(a) = 1; \textit{ and} \\ \lim_{n\to\infty} z_n(B,A) &> 0 \;\&\; \lim_{n\to\infty} z_n(A,B) > 0. \end{aligned}$$

*(2) When* $t < 1 - p$ *there is no responsive equilibrium and* $v^*(s) = 1$, $s = a, b$.

**Intuition** an individual signal has negligible impact on posterior beliefs in large committees and the single pivotal event under unanimity rule occurs when everyone else is voting to convict.

**Proof** [Sketch] Easy to see a symmetric responsive equilibrium must have $v^*(a) \in (0, 1)$ and $v^*(b) = 1$.

Hence, the probability an individual votes $B$ given $\omega = B$ is

$$P_B = p + (1 - p)v(a)$$

and the probability an individual votes $B$ given $\omega = A$ is

$$P_A = (1 - p) + pv(a).$$

Given an individual with signal $a$ is pivotal, Bayes rule implies indifference between voting $A$ or $B$ iff

$$\frac{(1-p)(P_B)^{n-1}}{(1-p)(P_B)^{n-1} + p(P_A)^{n-1}} = t.$$

Substituting and collecting terms,

$$v^*(a) = \frac{\left(\frac{(1-t)(1-p)}{tp}\right)^{\frac{1}{n-1}} p - (1-p)}{p - \left(\frac{(1-t)(1-p)}{tp}\right)^{\frac{1}{n-1}} (1-p)};$$

from which the theorem follows. □

F&P compute an example in which $p = 0.7$ and $t = 0.5$:

$$\begin{aligned}\lim_{n\to\infty} z_n(B,A) &= 0.22 \text{ and } z_{12}(B,A) = 0.21;\\ \lim_{n\to\infty} z_n(A,B) &= 0.47 \text{ and } z_{12}(A,B) = 0.48.\end{aligned}$$

F&P (1998) also provide a *full information equivalence* (FIE) result for non-unanimous rules, supporting the CJT for large committees with strategically rational agents (see also Myerson 1998).

Redefine (non-unanimous) $q$-rules such that $0 < q < 1$ and $B$ chosen iff strictly more than $nq$ voters vote $B$

### Theorem (Feddersen & Pesendorfer 1998)

*For all* $q < 1$,

$$\lim_{n\to\infty} \Pr[B|\ A, q, \mathbf{v}^*] = \lim_{n\to\infty} \Pr[A|\ B, q, \mathbf{v}^*] = 0.$$

Gerardi (2000) generalizes F&P's limit-FIE result for $q < 1$ to a model with private values and private information w.r.t. $t$, but finds that alternative $A$ (status quo) is almost always chosen in the limit when $q = 1$:

**Intuition** When there are private values, unlike in F&P, an individual with an $a$ signal might be pivotal under unanimity rule because the other $n - 1$ individuals are sufficiently low types as to vote for $B$ irrespective of their signal $[t < (1 - p)]$.

Given a strictly positive probability of "partisans", as $n \rightarrow \infty$ the proportion of voters voting informatively in equilibrium goes to zero, so the relative likelihood of the pivot event being due to low type partisans increases. $\square$

Duggan & Martinelli (2001) and Meirowitz (2002) generalize F&P to a continuum of signals.

# V Costly Information and Committee Design

Gersbach (1995) the first to consider costly information acquisition, assuming majority rule within a given committee and fully informative signals, suggesting that having all voters informed need not be socially efficient.

Persico (2004), Gerardi & Yariv (2008), Gershkov & Szentes (2009), Mukhopadhaya (2003), Li (2001) and Cai (2003) assume noisy signals and consider connections between committee size and the quality of the committee decision in various settings.

Suppose individuals can only acquire a noisy signal of the true state (with given precision, $p \in (\frac{1}{2}, 1)$) by paying a private cost $c > 0$.

Assume $c > 0$ sufficiently small that any individual is willing to purchase information if dictatorial.

**Free-rider problem** The greater the number of informed voters, the "better" is the collective decision likely to be but the smaller is the incentive for any individual to become informed.

(1) *What is the optimal committee size and voting rule?*

(2) *What is the optimal mechanism?*

Consider each question in turn.

# V(1) Optimal committee size and voting rule

Suppose a planner chooses

a committee of size $n$ from a large set of (ex ante) identical individuals; and
a $q$-rule, $q(n) \in \{1, \ldots, n\}$,

to maximize the common expected payoff from the collective choice (with lexicographic cost saving).

Having observed $(n, q)$, individuals simultaneously (and privately)

choose whether to become informed (observe a noisy signal) at cost $c > 0$;
vote *without* observing the information acquisition or votes of others.

If at least $q(n)$ votes are for $B$, then $B$ is implemented.

Recall: the statistical rule for any $n$ is $q^S(n) = k^*(n) + 1$, one more than the maximum number (from $n$) of "$b$" signals that any individual $i$ could observe and still prefer $A$ to $B$.

Let $(n^*, q^*)$ solve the planner's problem.

### Theorem (Persico 2004)

*(1) $q^* = q^S(n^*)$; (2) $n^*$ satisfies*

$$(1-p)p < \frac{q^S(n^*)}{n^*}\left[1 - \frac{q^S(n^*)}{n^*+1}\right].$$

**Intuition** Part *(1)*: All committee members become informed at the optimum. If not, the planner can choose a smaller committee and obtain the same expected payoff.

Since information is costly, an individual chooses to become informed only if it is subsequently a best response to vote informatively; informative voting is equilibrium behaviour under a $q$-rule iff $q = q^S$ (A-S&B 1996).

Part *(2)*: It turns out (Persico, Lemma 1) that (for non-trivial settings with binary signals) $q^S(n+2) = q^S(n) + 1$.

Since all committee members must choose to be informed and vote informatively at the optimal solution, the marginal incentive for each to acquire information given others do so must be positive at $n^*$ but not at $n^* + 2$. Otherwise, the planner, without compromising incentives for all to become informed and vote informatively, can increase committee size by two and the $q$-rule by one, thus improving expected payoff.

Applying the relevant incentive compatibility constraints yields the bound. $\square$

# V(2) Optimal mechanisms

simultaneous information acquisition

Persico (2004) assumes *informational decisions are simultaneous* and *limits attention to q-rules*; inter alia, these are necessarily ex post efficient but not necessarily ex ante optimal.

Gerardi & Yariv (2008) keep the first assumption but allow the planner to choose any (extended) mechanism $(n, \gamma)$, where $\gamma$ is the outcome function:

$$\gamma : \{\emptyset, a, b\}^n \rightarrow [0, 1].$$

Having observed $(n, \gamma)$, each committee member $i$ simultaneously (and privately)

chooses whether to become informed
sends a message $m_i \in \{\emptyset, a, b\}$ to the planner.

Given message profile $\mathbf{m}$, outcome $B$ chosen with probability $\gamma(\mathbf{m})$.

**Theorem (Gerardi & Yariv 2008)**

*The ex ante optimal mechanism can be ex post inefficient.*

From an ex ante, perspective, therefore, we can improve on Persico's solution if we allow ex post inefficient mechanisms that provide incentives for $n > n^*$ individuals to acquire information.

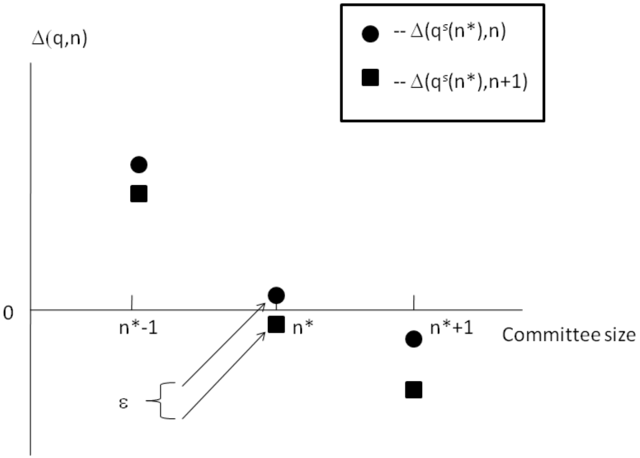
To do this, the outcome function $\gamma$ must 'waste' some information at some signal profiles, where the expected loss from the waste is less than the expected gain from having more people informed.

**Intuition** All (ex ante identical) committee members become informed and report truthfully at the optimum; so, wlog, the outcome function can be assumed anonymous:

Write $\gamma(k) = \Pr[B \text{ chosen} | \ \#\{s_i = b\} = k]$.

Suppose the optimal mechanism has $n^* + 1$ individuals become informed and suppose $c > 0$ small.

Let $\Delta(q, n)$ be the marginal incentive to acquire information, conditional on being pivotal in a size $n$ committee with $q = \min[k : \gamma(k) = 1]$.



Δ(q,n)
-- Δ(q^s(n*),n)
-- Δ(q^s(n*),n+1)
0
n*-1
n*
n*+1
Committee size
ε

Therefore, there exists a probability $\alpha \in (0,1)$ of choosing $B$ when $q^S(n^*)$ out of $n^*+1$ signals are "$b$", such that the individual is willing to become informed conditional on being pivotal with some probability; that is, choose $\gamma^*(k)$ such that

$$\gamma^*(k) = \begin{cases} 0 & \text{if } k < q^S(n^*) \\ \alpha & \text{if } k = q^S(n^*) \\ 1 & \text{if } k \geq q^S(n^*) + 1 \end{cases} . \square$$

# V(2) Optimal mechanisms

## sequential information acquisition

Gershkov & Szentes (2009) relax both the requirement that $\gamma$ must be a $q$-rule *and* the assumption that individual decisions must be made simultaneously.

Assume $\pi = 1/2$ and symmetric payoffs: $\forall i \in N$,

$u_i(x, \omega) =$

| | $A$ | $B$ |
|---|---|---|
| $A$ | 1 | 0 |
| $B$ | 0 | 1 |

The planner maximizes the difference, $|N|\, Eu - c\bar{n}$; where $\bar{n}$ is the expected number of citizens who acquire information and report signals (the de facto committee).

(G&S also allow for infinitely sized societies.)

Suppose that individuals are wholly non-strategic: they acquire and truthfully report signals if asked to do so by the planner.

Let $k(\mathbf{s})$ be the number of "*b*" signals in a sequence $\mathbf{s}$ of length $n(\mathbf{s}) \in \mathbb{N}$, and let

$$d(\mathbf{s}) = (n(\mathbf{s}) - k(\mathbf{s})) - k(\mathbf{s}),$$

be the difference between the numbers of "*a*" and "*b*" signals.

Say that the planner *makes the majority decision* (*maj*) if, for any sequence of messages $\mathbf{m}(\mathbf{s})$,

$$\gamma(\mathbf{m}(\mathbf{s})) = \begin{cases} 0 & \text{if } d(\mathbf{s}) > 0 \\ 1 & \text{if } d(\mathbf{s}) < 0 \end{cases};$$

and that the planner *continues (cont)* if an additional agent if asked to acquire information.

Relative to simultaneous mechanisms, sequential information gathering can yield cost efficiencies.

The first-best optimal mechanism involves sequential information acquisition and follows from "standard results in stochastic dynamic programming."

### Theorem (Gershkov & Szentes 2009)

*There exists a weakly decreasing step function* $g : \mathbb{N} \to \mathbb{N}$ *such that if, after asking* $n(\mathbf{s})$ *agents to acquire information, the reported signal sequence is* $\mathbf{s}$ *and* $|d(\mathbf{s})| \geq g(n(\mathbf{s}))$*, the planner makes the majority decision. Otherwise, the planner continues.*

In general, non-strategic behaviour is not incentive compatible.

G&S argue for focusing on ex post efficiency, despite potential conflict with ex ante optimality.

In an optimal mechanism, all those asked acquire information and report truthfully:

the planner selects an individual at random to acquire information and report the signal

if the planner's posterior belief exceeds a threshold, he makes a decision and otherwise continues; ...

incentive compatibility implies an individual becomes informed only if she could be pivotal; to induce such a belief:

(i) no individual is told either their position in the sequence or the reports of others

(ii) the threshold value for a final decision must be decreasing in the length of the sequence (else the marginal impact of an additional signal rapidly becomes negligible as the sequence extends).

Define a *state* to be a pair $\sigma = (n(\mathbf{s}), d(\mathbf{s}))$.

## Theorem (Gershkov & Szentes 2009)

*Suppose the first-best mechanism is not incentive compatible and let $\gamma^*$ be an ex ante optimal mechanism among those that are ex post efficient. Then $\gamma^*$ is characterized by a decreasing step-function $f : \mathbb{N} \to \mathbb{N}$ with $f(|N|) = 1$ and a set of states,*

$$\Sigma = \{\sigma : f(n(\mathbf{s})) = |d(\mathbf{s})|, f(n(\mathbf{s}) - 1) = f(n(\mathbf{s})) + 1\},$$

*such that:*
*(1) if $\sigma \notin \Sigma$ and $f(n(\mathbf{s})) \leq |d(\mathbf{s})|$ then $\Pr[maj|\ \sigma] = 1$;*
*(2) if $\sigma \notin \Sigma$ and $f(n(\mathbf{s})) > |d(\mathbf{s})|$ then $\Pr[cont|\ \sigma] = 1$;*
*(3) if $\sigma \in \Sigma$ then $\Pr[maj|\ \sigma] \geq 0$ and $\Pr[cont|\ \sigma] > 0$.*

G&S also show that, generically, there are at most two states in $\Sigma$ at which $\Pr[maj|\ \sigma] > 0$.

**Example** (G&S 2009) Let $N = \{1, \ldots, 9\}$, $p = 0.7$ and $c = 0.04$. Then $\gamma^*$ is such that:

$$f(n) = \begin{cases} 4 & \text{if } n \in \{1,2,3,4\} \\ 3 & \text{if } n \in \{5,6,7\} \\ 2 & \text{if } n = 8 \\ 1 & \text{if } n = 9 \end{cases} ;$$

and if

$$\sigma \neq (5,3) \;\&\; \sigma : \begin{cases} |d(\mathbf{s})| \geq f(n(\mathbf{s})) & \text{then } \Pr[maj|\sigma] = 1 \\ |d(\mathbf{s})| < f(n(\mathbf{s})) & \text{then } \Pr[cont|\sigma] = 1 \end{cases} ,$$

$$\sigma = (5,3) \text{ then } \Pr[maj|\sigma] = 1 - \Pr[cont|\sigma] \in (0,1).$$

At $(n(\mathbf{s}), d(\mathbf{s})) = (5, 3)$ there are 4 "*a*" signals and 1 "*b*" signal.

Therefore, if the remaining 4 individuals acquire information and observe "*b*" signals, the majority decision would be reversed.

But this is a relatively low probability event, so the randomization at $\sigma = (5, 3)$ is necessary to preserve incentive compatibility for the 5th person to acquire information. $\square$

Koriyama & Szentes (2007) ask about welfare costs of non-optimally sized committees and find that, in model with an arbitrary large number of signals and a message space sufficiently large to support full revelation, any larger-than-optimal committee aggregates information more efficiently than a committee two members smaller-than-optimal.

# VI Optimal Voting Rules with Costless Information

Chwe (2008) considers a size $n$ (odd) committee with the usual binary signal structure but

heterogeneous preferences: $\forall i$, $u_i(A, A) > u_i(A, B)$ and $u_i(B, B) > u_i(B, A)$, and

heterogeneous priors: $\forall i$, $i$'s prior $\Pr[\omega = A] = \pi_i \in (0, 1)$

Individual *i*'s *bias* for *A* is

$$\phi_i(A) = \frac{\pi_i[u_i(A,A) - u_i(A,B)]}{\pi_i[u_i(A,A) - u_i(A,B)] + (1-\pi_i)[u_i(B,B) - u_i(B,A)]}$$

and $\phi_i(B) = [1 - \phi_i(A)]$.

*i* is *unbiased* if $\phi_i(x) = 1/2$.

Unde the *supermajority penalty* (SP) mechanism, an alternative is

surely chosen if the number of signals in its favour is small or a weak majority

surely rejected if the number of signals in its favour is large

chosen with a given probability at the (well-defined) switch point between large and small.

### Theorem (Chwe 2008)

*Suppose there exist $i, j$ such that $\phi_i(a) < 1 - p$ and $\phi_j(b) > p$. Then, with respect to an unbiased individual, the optimal anonymous, neutral and incentive compatible mechanism is the SP mechanism. If there are no such individuals, then the optimal anonymous, neutral and incentive compatible mechanism is majority rule.*

Non-monotonicity insures incentive compatibility among committee members with strong biases for an alternative:

If the rule is monotonic, $\phi_i(b) > p$ implies voting $B$ irrespective of $s_i$ is a best response; non-monotonicity implies a pivot event at which an additional $B$ vote yields outcome $A$ (see also Chwe 1999).

**Example** (Chwe 2008) Suppose $n = 7$, signal precision $p = \frac{2}{3}$ and there exist partisans $i, j$.

Let $V_B$ be the number of votes for $B$.

Then the optimal anonymous, neutral and incentive compatible mechanism is such that,

$$\begin{aligned} \text{Pr}[\text{choose } B | V_B &\in \{0, 4, 5\}] = 1 > \\ \text{Pr}[\text{choose } B | V_B &= 1] = (1 - \text{Pr}[\text{choose } B | V_B = 6]) > \\ \text{Pr}[\text{choose } B | V_B &\in \{2, 3, 7\}] = 0. \ \square \end{aligned}$$

Ben-Yashar & Milchtaich (2004) take an alternative approach in which voters receive different quality signals and anonymity is not imposed: in this setting, the optimal rule is a weighted voting rule with weights reflecting asymmetries in the precision of individuals' signals.

# VII Sequential Voting with Costless Information

Ordeshook & Palfrey (1988) are the pioneers ...

Dekel & Piccione (2000) address this question in a symmetric environment with private values and binary signals.

## Theorem (Dekel & Piccione 2000)

*An informative strategy profile is a symmetric equilibrium under simultaneous voting if and only if it is a sequential equilibrium for any sequential voting protocol.*

Conditioning on being pivotal yields all of the inferable decision-relevant information in any symmetric equilibrium, irrespective of timing.

Additional asymmetric equilibria exist under sequential games, however, that are unavailable in the simultaneous setting (see also Wit 1997 and Fey 1996).

Battaglini (2006) shows the Dekel & Piccione result is not robust to the introduction of an arbitrarily small cost of voting; in this setting, the equilibrium sets of sequential and simultaneous voting protocols are disjoint:

as with costly information acquisition, when voting is costly the relative magnitude of pivot probabilities directly affects individuals incentives to vote, so limiting opportunities for information aggregation.

# VIII Some Questions To Be Considered

What happens as $n$ becomes large?

What happens if people can abstain?

What happens if people can talk before voting?

What do people *really* do when voting under incomplete information?!