

# The Design of School Choice Clearinghouses: NYC and Boston

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# Today's talks

## NYC Schools (Roth):

- design of a centralized high school allocation procedure, implemented in 2003-04, for students entering Sept. '04

## Boston Schools (Pathak):

- redesign of a school allocation procedure, implemented for students entering grades K, 6, and 9 in September 2006.

# Economists As Engineers

- In recent years, game theorists have become usefully involved in the design of markets. E.g.
  - Labor markets (doctors and other health professions)
  - Auctions
  - Kidney Exchange and other organ transplants
  - School matching (NYC, Boston)
- A certain amount of humility is called for: these things consist of complex, overlapping institutions, and there are often subtle social and political goals.
  - The role of the economist is to help make sure that whatever is being done is being done efficiently, in a way that makes participation safe and simple.

# A general market design framework to keep in mind:

- To achieve efficient outcomes, marketplaces need make markets sufficiently
  - **Thick**
    - Enough potential transactions available at one time
  - **Uncongested**
    - Enough time for offers to be made, accepted, rejected, transactions carried out...
  - **Safe**
    - Safe to participate, and to reveal relevant preferences
- Some kinds of transactions are **repugnant**...and this can constrain market design.

# The **Old** New York City High School Choice System

## **Decentralized** application and admission

- **congested**: left **30,000** kids each year to be administratively assigned (while about 17,000 got multiple offers)
  - **Waiting lists run by mail**
  - Gaming by high schools; **withholding of capacity**
  - **Unsafe for families to truthfully reveal preferences**
- The new mechanism is a **centralized clearinghouse** that produces **stable matches**.
  - We now have enough data to begin to say something about how it is working.

# Old NYC High School Match

(Abdulkadiroglu, Pathak, Roth 2005, 2009)

## Overview:

- Over 90,000 students enter high school each year in NYC
- Each was invited to submit list of up to 5 choices
- Each student's choice list distributed to high schools on list, who independently make offers
- Only approx. 40% of students receive initial offers, the rest put on waiting lists—3 rounds to move waiting lists...
- Approx. **30,000** students assigned to schools not on their choice list.

## Issues in old (2002) system

- Schools see rank orders
  - Some schools take students' rankings into account & consider only those that rank their school first
  - So a student whose second choice school will only consider those who list it first has a difficult choice, especially if the true first choice is hard to get into
- Students need to strategize (not safe to reveal true preferences):
  - The 2002-03 *Directory of the NYC Public High Schools* : “determine what your competition is for a seat in this program”

# Issues in old (2002) system

- Principals concealing capacities  
Deputy Chancellor (NYT 11/19/04):  
“Before you might have had a situation where a school was going to take 100 new children for 9<sup>th</sup> grade, they might have declared only 40 seats and then placed the other 60 children outside the process.”

# Issues in old (2002) system

## 1. “5” choices

- 52% of kids rank five choices → constraint binding
- Congestion, nevertheless (Roth and Xing, 1997): Not enough offers and acceptances could be made to clear the market
- Only about 50,000 out of 90,000 received offers initially.
- About 30,000 assigned outside of their choice

## 2. Multiple offers—are they good for some kids?

- about 17,000 received multiple offers
- Students may need time to make up their mind, especially if we want to keep desirable students from going to private school
- Only 4% don't take first offer in 02-03 at the cost of over 30,000 kids not getting any offer

# NYC School System

The high schools are organized to achieve a number of social goals

	# of Programs
Unscreened (no preferences)	86
Screened & Auditioned	188
Specialized HS	6
Educational Option (no preferences for half seats)	252

In Brooklyn, Bronx, Manhattan, and Staten Island

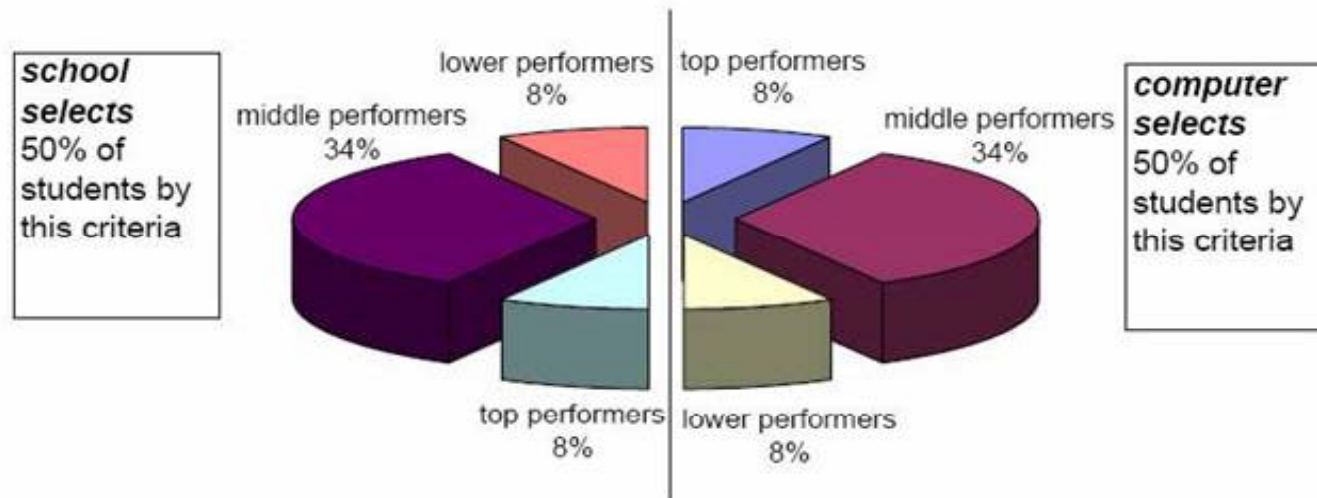
Unscreened capacity largest

Roughly 25,000 kids take Specialized High School Test

# NYC School System

**Ed-Opt Schools** – based on city or state standardized reading test score grade 7  
(preferences for only half the seats)

Educational Option Program Student Selection Breakdown



Are NYC Schools a *two-sided* market?  
i.e. are the actors just families, or are the schools  
also strategic actors?

Two facts:

1. Schools conceal capacities  
i.e. principals act on instabilities
2. Principals of different EdOpt schools  
have different preferences even over  
the students with the lowest scores,  
some preferring higher scores, some  
preferring better attendance records

# A (too) simple basic model

- Schools =  $\{s_1, \dots, s_n\}$       Children =  $\{c_1, \dots, c_p\}$   
# positions  $q_1, \dots, q_n$
- PREFERENCES (complete and transitive):  
 $P(s_i) = c_3, c_2, \dots, s_i$      $[c_3 P(s_i) c_2]$  (not all strict)  
 $P(c_j) = s_2, s_4, \dots, c_j, \dots$
- An OUTCOME of the game is a *MATCHING*:  
 $\mu: S \cup C \rightarrow S \cup C$   
such that  $\mu(s) = c$  iff  $\mu(c) = s$ , and for all  $s$  and  $c$   
 $|\mu(s)| \leq q_s$ , and  
either  $\mu(c)$  is in  $S$  or  $\mu(c) = c$ .

# Stable matchings

A matching  $\mu$  is

- *BLOCKED BY AN INDIVIDUAL*  $k$  if  $k$  prefers being single to being matched with  $\mu(k)$ , i.e.  $k \succ_k \mu(k)$   
( $\mu(k)$  is *unacceptable*).
- *BLOCKED BY A PAIR OF AGENTS*  $(s,c)$  if they each prefer each other to  $\mu$ , i.e.
  - $c \succ_s \mu(s)$  and  $s \succ_c \mu(c)$
- A matching  $\mu$  is *STABLE* if it isn't blocked by any individual or pair of agents.

# Basic Deferred Acceptance (Gale and Shapley 1962)

- Step 0.0: students and schools **privately** submit preferences
- Step 0.1: arbitrarily break all ties in preferences (via a lottery)
- Step 1: Each student “applies” to her first choice. Each school tentatively assigns its seats to its applicants one at a time in their priority order. Any remaining applicants are rejected.
- ...
- Step k: Each student who was rejected in the previous step applies to her next choice if one remains. Each school considers the students it has been holding together with its new applicants and tentatively assigns its seats to these students one at a time in priority order. Any remaining applicants are rejected.
- The algorithm terminates when no student application is rejected, and each student is assigned her final tentative assignment.

# Theorems (for the simple model)

1. The outcome that results from the student proposing deferred acceptance algorithm is stable, and (when preferences are strict) student optimal among the set of stable matchings (Gale and Shapley, '62)
2. The student proposing deferred acceptance algorithm makes it a **dominant strategy for students to state their true preferences.** (Dubins and Friedman 1981, Roth, 1982, 1985)
  1. This means it is safe for families to submit their true preferences: e.g. even if they don't get their true first choice, it won't hurt their chance of getting their second choice.

# The New (Multi-Round) Deferred Acceptance Algorithm in NYC

- We advised, *sometimes convinced*, the NYC Department of Education
- Software and the online application process has been developed by a software consulting company
- The new design adapted to the regulations and customs of NYC schools

Some (Imperfectly Resolved)  
Design issues  
(It's important to choose your fights:)

# Redesign: 12 choice constraint

- DOE thought this would be sufficient, we encouraged more

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## New Process: Average Number of Rankings Each Round

Round	Ranking											
	1	2	3	4	5	6	7	8	9	10	11	12
Round 1	91,286 100%	84,554 93%	79,646 87%	73,398 80%	66,724 73%	59,911 66%	53,466 59%	47,939 53%	42,684 47%	37,897 42%	31,934 35%	22,629 25%
Round 2	87,810 100%	81,234 93%	76,470 87%	70,529 80%	64,224 73%	57,803 66%	51,684 59%	46,293 53%	41,071 47%	35,940 41%	29,211 33%	18,323 21%
Round 3	8,672 100%	8,139 94%	7,671 88%	7,025 81%	6,310 73%	5,668 65%	5,032 58%	4,568 53%	4,187 48%	3,882 45%	3,562 41%	3,194 37%

3,476 Specialized High Schools Students  
91,286 Total students

# Partial incentive compatibility for constrained choosers

- **Proposition** (Haeringer and Klijn, Lemma 8.1.): In the student-proposing deferred acceptance mechanism where a student may only rank  $k$  schools,
  - if a student prefers fewer than  $k$  schools, then she can do no better than submitting her true rank order list,
  - if a student prefers more than  $k$  schools, then she can do no better than employing a strategy which selects  $k$  schools among the set of schools she prefers to being unassigned and ranking them according to her true preference ordering.

# Multiple Rounds

- Historical/legal constraints: difficult to change specialized high school process/cannot force a student who gets an offer from a specialized high school to take it
  - Round 1: run algorithm with all kids in round 1, not just specialized students; only inform specialized students
    - Unstable if a specialized kid does not get a spot at a non-specialized high school when considered at round 1, but could get that spot in round 2
  - May not a big problem if students with specialized high schools offers are ranked high in all schools' preferences, and/or if most students prefer to go to a specialized school
    - In old system, ~70% of kids with an offer from a specialized program took it, 10% of kids went to private school and 14% kids went to either their first or second choice from the other schools.
      - Potential instabilities among these 14% will not be large if they are also considered highly desirable by the non-specialized schools they apply to.
      - ... (however, we do observe several hundred children who decline<sub>2,1</sub> a specialized school for their not-top-choice mainstream school...)

# Multiple Rounds

- Need to assign unmatched kids; unlike medical labor markets everyone must go to school
  - Round 3
    - “No time” for high schools to re-rank students in round 3, so no new high school preferences expressed
      - Another place where random preferences are used for some screened schools.

## Lotteries: Equity and perception

How should we rank students in schools that do not have preferences over students?

- For unscreened schools and in round 3
  - A single lottery that applies to each school?
  - Or a different lottery for every such school?
- A single lottery avoids instabilities that are due to randomness (Abdulkadiroglu & Sonmez, 2003)

# Lotteries, cont.:

## Explaining and defending

NYC DOE argued that a more equitable approach would be to draw a new random order for each school:

Here are some of the emails we got on the subject:

- “I believe that the equitable approach is for a child to have a new chance... This might result in both students getting their second choices, the fact is that each child had a chance. If we use only one random number, and I had the bad luck to be the last student in line this would be repeated 12 times and I never get a chance. **I do not know how we could explain that to a student and parent.**”
- **“When I answered questions about this at training sessions, (It did come up!) people reacted that the only fair approach was to do multiple runs.”**

## Lottery, cont.

- Ran simulations. These simulations showed that the efficiency loss due to multiple draws was considerable; and increases with correlation in students' preferences.
- We pushed hard on this one, but it looked like the decision was going to go against us. But we did get the NYC DOE to agree to run the algorithm both ways and compare the results on the submitted preference lists.
- They agreed, and eventually decided on a **single rank order** after seeing welfare gains on the submitted preferences

# Tie-breaking in Student-Proposing Deferred Acceptance in the First Round 2003-04

Choice	Number Ranking	Single Tie-Breaking (250 draws)	Multiple Tie-Breaking (250 draws)	
1	5,797 (6.7%)	21,038 (24.82%)	19,783 (23.34%)	No stochastic dominance
2	4,315 (5.0%)	10,686 (12.61%)	10,831 (12.78%)	
3	5,643 (6.6%)	8,031 (9.48%)	8,525 (10.06%)	
4	6,158 (7.2%)	6,238 (7.36%)	6,633 (7.83%)	
5	6,354 (7.4%)	4,857 (5.73%)	5,108 (6.03%)	
6	6,068 (7.1%)	3,586 (4.23%)	3,861 (4.56%)	
7	5,215 (6.1%)	2,721 (3.21%)	2,935 (3.46%)	
8	4,971 (5.8%)	2,030 (2.40%)	2,141 (2.53%)	
9	4,505 (5.2%)	1,550 (1.83%)	1,617 (1.91%)	
10	5,736 (6.7%)	1,232 (1.45%)	1,253 (1.48%)	
11	9,048 (10.5%)	1,016 (1.20%)	894 (1.05%)	
12	22,239 (25.8%)	810 (0.96%)	372 (0.44%)	
unassigned	-	20,952 (24.72%)	20,795 (24.54%)	26

# First Year of Operation

- Over 70,000 students were matched to one of their choice schools
  - an increase of more than 20,000 students compared to the previous year match
- An additional 7,600 students matched to a school of their choice in the third round
- 3,000 students did not receive any school they chose
  - 30,000 did not receive a choice school in the previous year

# First year, cont

Much of the success is due to

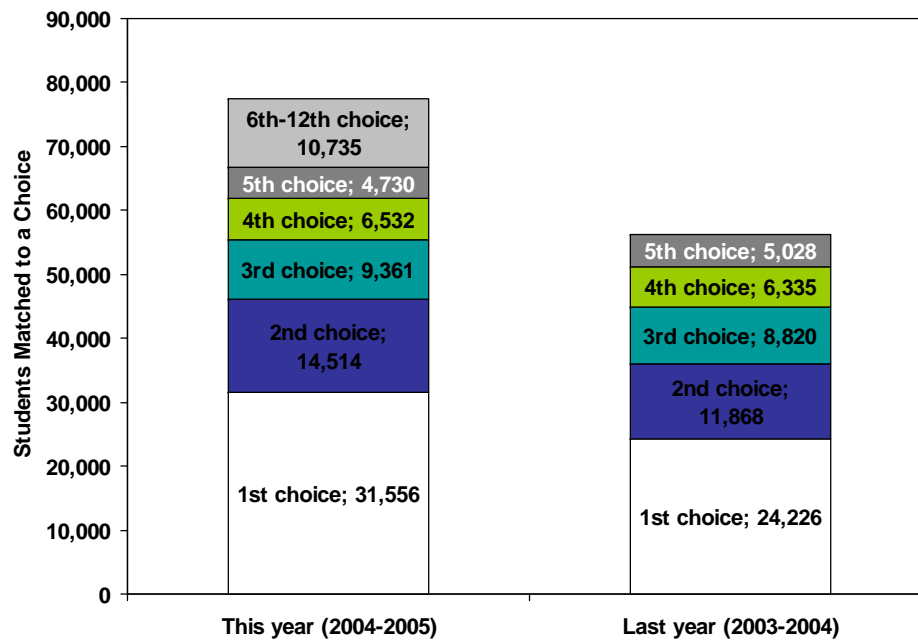
- relieving congestion
  - Allowing many offers and acceptances to be made, instead of only 3
  - giving each student a single offer rather than multiple offers to some students
- allowing students to rank 12 instead of 5 choices
- But more than that is going on...

# First year results:

## More students get top choices

(this is a chart prepared by NYCDOE, comparing academic years 04-05 and 03-04)

### Number of students matched at the end of Round II



- **21,000** more students matched to a school of their choice
- **7,000** more students receiving their first choice
- **10,000** more students receiving one of their top 5 choices

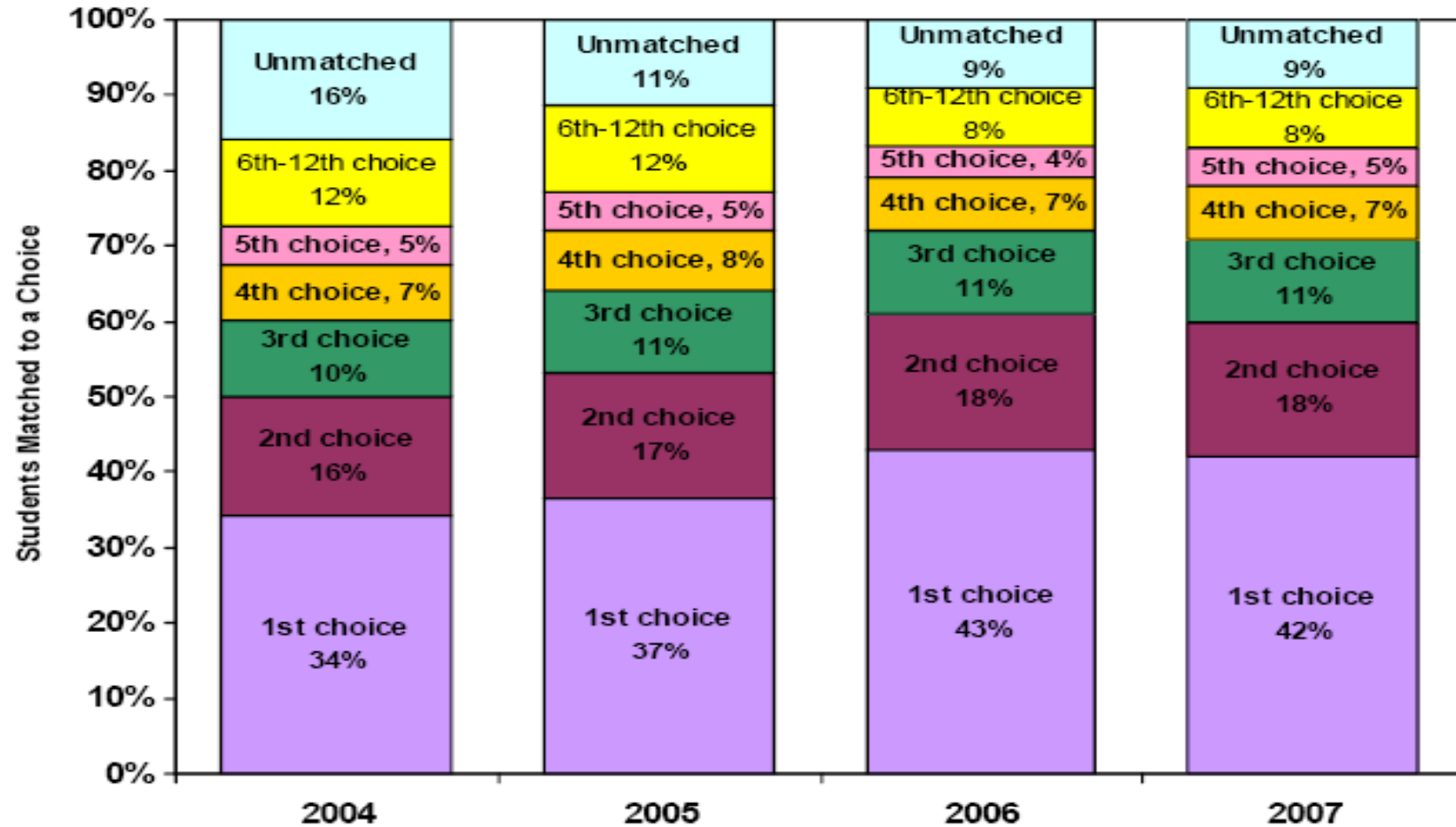
# The results show continued improvement from year to year

- Even though no further changes have been made in the algorithm...

# First 4 years: March 23, 2007

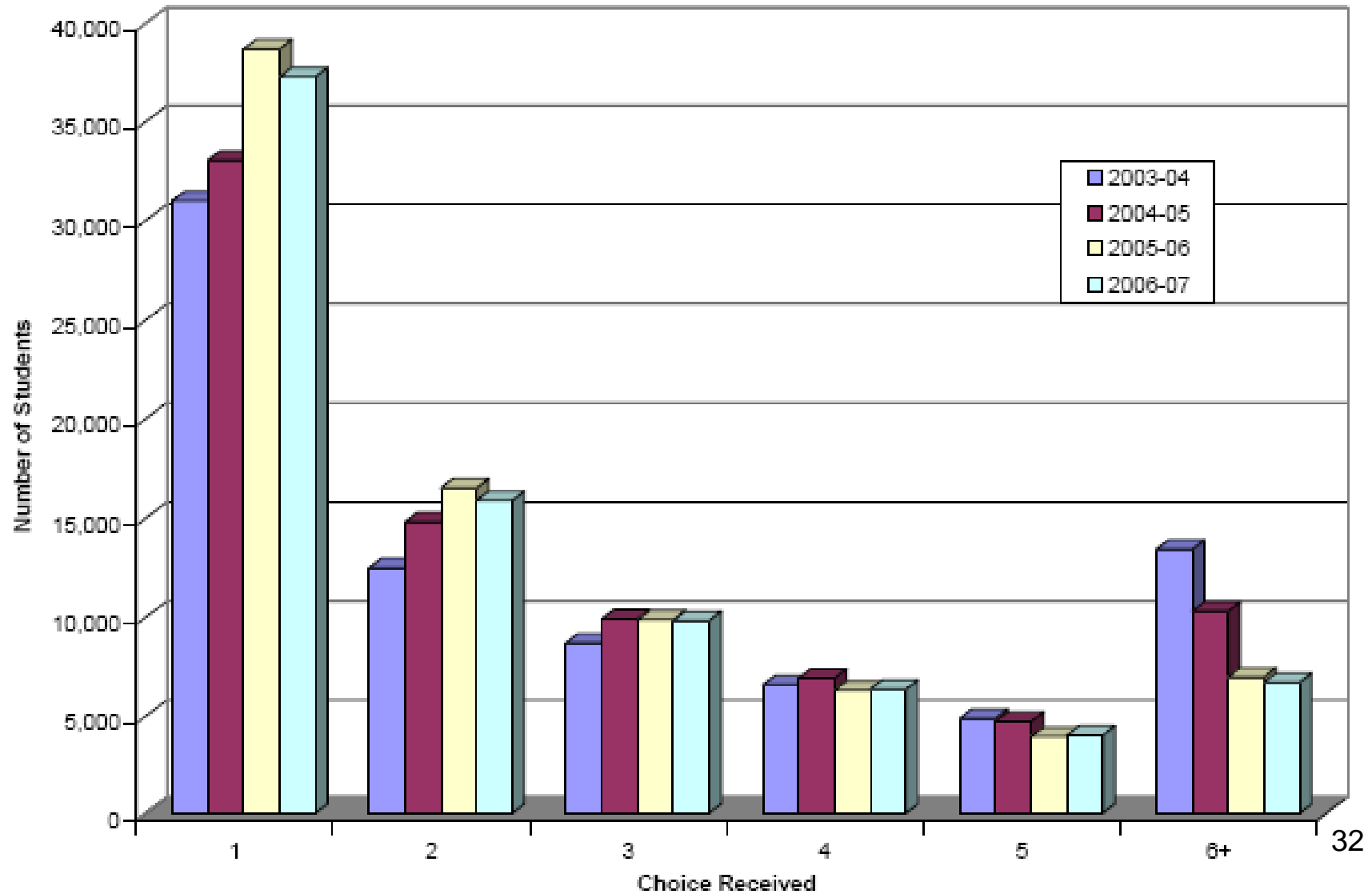
## Results at end of Round 2

(Schools have learned to change their reporting of capacities)



# What happened in NYC after the algorithm was introduced in 2003-04?

Figure 1: Distribution of Choices Received in Round 1 or 2 by Year



# What is going on?

- It appears that schools are no longer withholding capacity.
- Some high schools (even top high schools like Townsend Harris) have learned to rank substantially more than their capacity, because many of their admitted students go elsewhere (e.g. admissions to Townsend Harris provides good leverage for bargaining over financial aid with private schools).
- This allows more students to be accepted to their top choice, second choice, etc. during the formal match process.

# Immediate Issue: Appeals

- Just over 5,100 students appealed in the first year
- Around 2,600 appeals were granted
- About 300 of the appeals were from students who received their first choice
- Designing an efficient appeals process—top trading cycles?
  - A dry run in year 2 showed that many students could be granted appeals without modifying school capacities.
    - One 40-student cycle...
- Lately (2006-08) TTC was used
  - One 26 student cycle

# The old appeals system

- Administrative, child by child decisions...
- Changes in school capacities; i.e. schools were filled to over capacity...
- New approach: centralized clearinghouse for appeals...
  - Allows more appeals to be granted without violating the capacity constraints of schools.

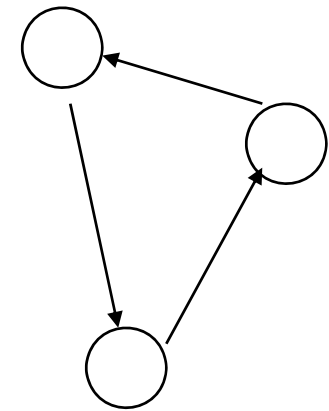
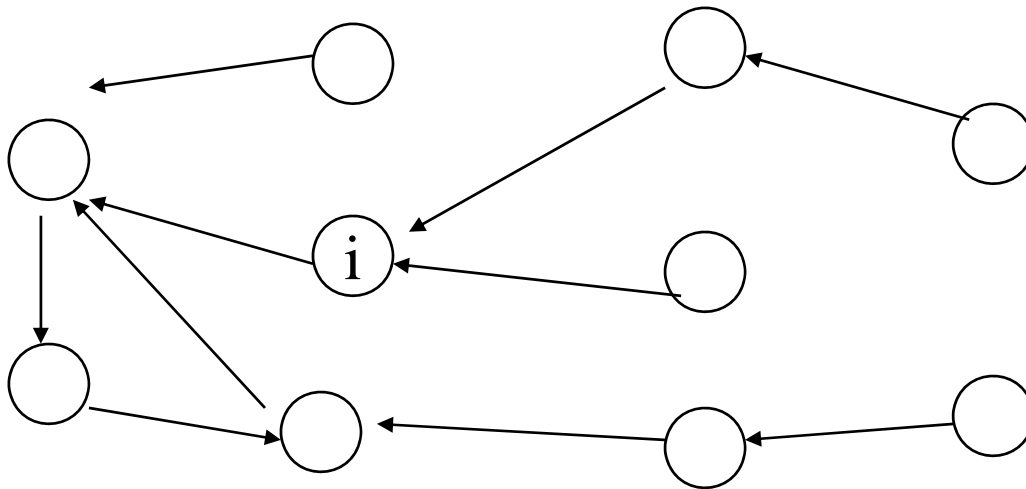
## A slightly too simple 1-sided model:

- $n$  students each with a currently assigned school place.
- Each student has preferences over schools.
- Gale's top trading cycles (TTC) algorithm: Each student points to her most preferred school (and each school place points to its current occupant). There is at least one cycle in the resulting directed graph (a cycle may consist of an agent pointing to her own place.) In each such cycle, the corresponding trades are carried out and these students are removed from the market together with their new assignments.
- The process continues (with each student pointing to her most preferred place that remains available) until no students and places remain.

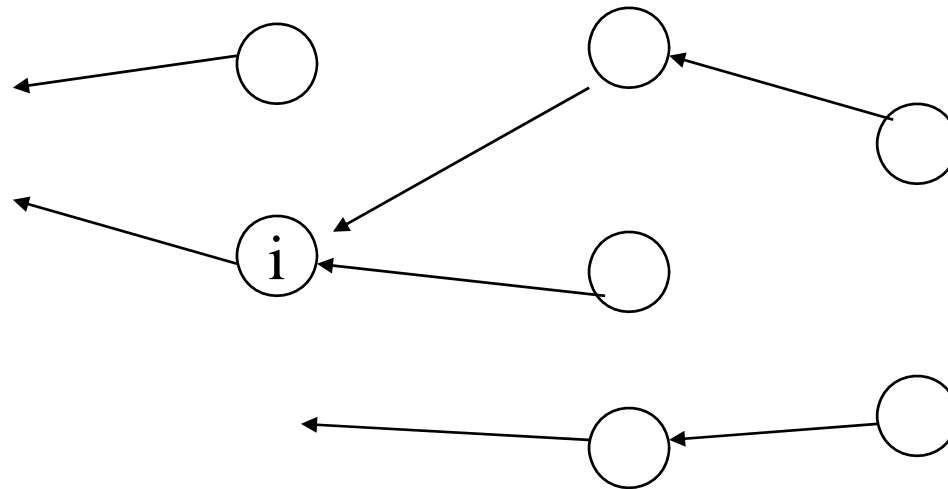
Theorem (Roth 1982): if the top trading cycle procedure is used, it is a dominant strategy for every agent to state his true preferences.

- So this is a procedure that makes it safe and simple for students to participate
- The *idea* of the proof is simple, but it takes some work to make precise.

# Cycles and chains



The cycles leave the system (regardless of where  $i$  points), but  $i$ 's choice set (the chains pointing to  $i$ ) remains, and can only grow



# NYC--summary

- **Waiting lists** are a congested allocation mechanism—congestion leads to instabilities and strategic play.
- NYC high schools—only recently re-centralized—are active players in the system.
- Information about the mechanism is part of the mechanism.
  - Information dissemination within and about the mechanism is part of the design
- New mechanisms can have both immediate and gradual effects.
- Appeals may be a big deal
  - when the preferences are those of 13 and 14 year olds
  - When a nontrivial percentage of assigned places aren't taken up because of withdrawals from the public school system (moves, and private schools)
- Open question:
  - How best to design appeals, in light of changing preferences of 13 year olds, mobile school population, but to continue to give good incentives in the main match?

# Matching mechanisms more generally

- While there are some general principles, local details matter.
- Different kinds of matching problems need different kinds of clearinghouses.

# U.S. Medical Labor Market Clearinghouses (Roth-Peranson )

## NRMP / SMS:

- Medical Residencies in the U.S. (NRMP) (1952)
- Abdominal Transplant Surgery (2005)
- Child & Adolescent Psychiatry (1995)
- Colon & Rectal Surgery (1984)
- Combined Musculoskeletal Matching Program (CMMP)
  - Hand Surgery (1990)
- Medical Specialties Matching Program (MSMP)
  - Cardiovascular Disease (1986)
  - **Gastroenterology (1986-1999; rejoined in 2006)**
  - Hematology (2006)
  - Hematology/Oncology (2006)
  - Infectious Disease (1986-1990; rejoined in 1994)
  - Oncology (2006)
  - Pulmonary and Critical Medicine (1986)
  - Rheumatology (2005)
- Minimally Invasive and Gastrointestinal Surgery (2003)
- Obstetrics/Gynecology
  - Reproductive Endocrinology (1991)
  - Gynecologic Oncology (1993)
  - Maternal-Fetal Medicine (1994)
  - Female Pelvic Medicine & Reconstructive Surgery (2001)
- Ophthalmic Plastic & Reconstructive Surgery (1991)
- Pediatric Cardiology (1999)
- Pediatric Critical Care Medicine (2000)
- Pediatric Emergency Medicine (1994)
- Pediatric Hematology/Oncology (2001)
- Pediatric Rheumatology (2004)
- Pediatric Surgery (1992)

## Primary Care Sports Medicine (1994)

## Radiology

- Interventional Radiology (2002)
- Neuroradiology (2001)
- Pediatric Radiology (2003)

## Surgical Critical Care (2004)

## Thoracic Surgery (1988)

## Vascular Surgery (1988)

## Postdoctoral Dental Residencies in the United States

- Oral and Maxillofacial Surgery (1985)
- General Practice Residency (1986)
- Advanced Education in General Dentistry (1986)
- Pediatric Dentistry (1989)
- Orthodontics (1996)

## Psychology Internships in the U.S. and CA (1999)

## Neuropsychology Residencies in the U.S. & CA (2001)

## Osteopathic Internships in the U.S. (before 1995)

## Pharmacy Practice Residencies in the U.S. (1994)

## Articling Positions with Law Firms in Alberta, CA(1993)

## Medical Residencies in CA (CaRMS) (before 1970)

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# Kidney Exchange Clearinghouses

New England Program for Kidney Exchange (2005)  
Alliance for Paired Donation (2006)  
U.S. National Match (2010?)



# Belgian Schools will have special needs

- Some of their problems will resemble those of NYC and Boston
- But careful attention will need to be paid to the particular needs of Belgian schools.
- Estelle Cantillon is a market designer who is in a good position to know both the latest details of market design, and who understands the local context.

# Market design for school choice

- Thickness
  - In both NYC the market became thicker once schools no longer withheld some places.
- Congestion
  - In NYC, congestion was the most visible problem of the old system, which led to problems of safe participation (and thickness)
  - In Boston there was already a centralized mechanism in place
- Safety
  - In NYC, there were both participation problems and incentive problems about revealing preferences.
  - In Boston, the big problem was about revealing preferences

# Background reading

- **New York City**
- Abdulkadiroglu, Atila , Parag A. Pathak, and Alvin E. Roth, "[The New York City High School Match](#)," *American Economic Review, Papers and Proceedings*, 95,2, May, 2005, 364-367.
- Abdulkadiroglu, Atila , Parag A. Pathak, and Alvin E. Roth, "[Strategy-proofness versus Efficiency in Matching with Indifferences: Redesigning the NYC High School Match](#)," revised, November, 2008, *American Economic Review*, forthcoming.
- **Boston**
- Abdulkadiroglu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sonmez, "[The Boston Public School Match](#)," *American Economic Review, Papers and Proceedings*, 95,2, May, 2005, 368-371.
- Abdulkadiroglu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sonmez, "[Changing the Boston School Choice Mechanism](#)," May 2006.
- **Market Design in general**
- Roth, Alvin E. "[What have we learned from market design?](#)" Hahn Lecture, *Economic Journal*, 118 (March), 2008, 285-310.

Now I turn the stage over to  
Parag Pathak

