Monopoly and Oligopoly in homogeneous product markets.

TA session 1
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Roadmap: Homogeneous Products

- The aggregate demand
- Static Monopoly: (Ch.1 Tirole)
- Oligopoly theory in Homogeneous product industries: (section II Ch.5 Tirole)
- Non competitive behavior: Collusion (section II Ch.6 Tirole)
The aggregate demand

- Microeconomic theory starts from individual preferences and through the utility maximization problem, finds the individual (walrasian) demand function $x(p,w)$.

- The properties of $x(p,w)$ are well known (HD0 in $(p,w)$, Walras’ law, convexity and uniqueness, Roy’s identity, …).

- In IO we usually focus on a few products (partial analysis) and collapse all other goods into a generic outside good.

- Suppose we focus on one good, good 1. Summing up all the individual demands of consumers we obtain the aggregate demand for good 1. Suppose that there are $I$ consumers, each of them with rational preference relations this must be:

$$D_1(p_1, p_0, w_1, ..., w_I; Z) = \sum_{i} x_{i1}(p_1, p_0, w_i; Z)$$

- This aggregate demand depends not only on prices but also on the specific wealth levels of the various consumer as well as other demand shifters

- When we estimate demand in IO we usually make functional form assumptions on $D$. It is important to keep in mind that such assumptions must be consistent with individual demand
The aggregate demand

- As an example with differentiated products, consider Hotelling’s model of horizontal differentiation (partial analysis focusing on two goods)

- Hotelling proposes to consider a linear city of length 1, with consumer distributed uniformly along the city. Two shops, located at the two ends of the city, sell the same physical good. For some characteristics, the optimal choice – at equal prices – depends on the particular consumer and tastes vary in the population.

- Consumer have unit demands: consume zero or one unit of the good, and they must pay a transportation cost to go to the shop.

- The individual demand will depend on the price difference charged by the two shops and on the transportation cost.

- If the price difference between the two shops does not exceed transportation cost along the whole city and if prices are “not too high” there exist a consumer that has a location that make is indifferent between buying from shop 1 or 2.

- Then consumers can be aggregated respect to their location, and we can find the demand faced by each shop.
The aggregate demand
The aggregate demand

- The generalized price of going to shop 1 is $p_1 + tx$.

- The generalized price of going to shop 2 is $p_2 + t(1-x)$.

- If $\hat{s}$ denotes the surplus enjoyed by each when he is consuming the good, then the utility of the consumer located at $x$ is $\hat{s} - p_1 - tx$ if he buys at shop 1, $\hat{s} - p_1 - t(1-x)$.

- If $p_2 - p_1 < t$ and prices are not too high there exist a consumer with a location $x^*$ who is indifferent between buying from shop 1 and buying from shop 2:

  $$p_1 + tx^* = p_2 + t(1-x^*) \iff x^*(p_1,p_2) = \frac{p_2 - p_1 + t}{2t}$$

  Then the demands are $D_1(p_1,p_2)=N x^*(p_1,p_2)$ and $D_2(p_1,p_2)=N[1-x^*(p_1,p_2)]$

- If $p_2 - p_1 > t$, shop 2 has no demand. Shop 1 has demand $D_1(p_1,p_2)=N$, if $p_1 \leq \hat{s} - t$

  and demand $D_1(p_1,p_2)=N(\hat{s} - p_1)$, if $p_1 > \hat{s} - t$

- The third case is that every shop has a local monopoly power and the market is not covered.
The aggregate demand

- Why do we care about aggregate demand?

It allows us to solve the firm’s profit maximization problem and to find equilibrium prices and quantities, and thus perform all kinds of welfare analysis or counterfactual policy exercises.
Monopoly theory

- Main framework
- Inverse elasticity rule
- Second order conditions
- The revenue function
- Properties of monopoly price
- The Deadweight loss
Monopoly theory – Main Framework

• Assumptions:

1. Goods produced by monopolist are given

2. Monopolist charges the same price per unit of good for each good produced.
Monopoly theory – Inverse Elasticity Rule

Define:

\( q = D(p) \), Demand \( D'(\cdot)<0 \); Demand is differentiable and decreasing in price.
\( p = P(q) \), Inverse Demand with \( P'(\cdot)<0 \); Price is decreasing in quantity demanded.
\( C(q) \), Differentiable cost function, \( C'(\cdot)>0 \) is differentiable and increasing in output.

The Monopolist’s Profit Maximization Problem hence is:

\[
\text{Max}_{(p,q)} \ p \cdot q - C(q)
\]
\[
s. \ t. \ q = D(p)
\]

Plugging in the constraint we get rid of one unknown:

\[
\text{Max}_{(p)} \ p \cdot D(p) - C(D(p))
\]

F.o.c. \( (p) \):

\[
D(p) + p \cdot D'(p) - c'(D(p)) \cdot D'(p) = 0
\]
Monopoly theory – Inverse Elasticity Rule

By rearranging and defining $p^m$ as the monopolist price we get:

$$p^m - c'(D(p^m)) = -\frac{D(p^m)}{D'(p^m)}$$

Rewriting the equation, we isolate the Lerner index (left side):

$$\frac{p^m - c'(D(p^m))}{p^m} = -\frac{D(p^m)}{p^m} \cdot \frac{1}{D'(p^m)}$$

Furthermore:

$$\frac{1}{\varepsilon} = -\frac{D(p^m)}{p^m} \cdot \frac{1}{D'(p^m)}$$

• The Lerner index measures the market power of the firm, i.e. the ability of the firm to maintain the price above the marginal cost.

• In perfect competition the Lerner Index is zero.
Monopoly theory – Second order conditions

Then we compute second order conditions to check that our solution correspond to a maximum:

\[ \text{S.o.c. (p): } 2 \cdot D'(p) + p \cdot D''(p) - C''(D(p)) \cdot [D'(p)]^2 - D''(p) \cdot C'(D(p)) < 0, \text{ for every } p \]

Then Foc is sufficient condition.
Monopoly theory – The Revenue Function

Equivalently, let $q^m = D(p^m)$ be the monopolist output we can solve the problem w.r.t. monopoly quantities:

$$\max_{q} P(q) \cdot q - C(q)$$

F.o.c. $(q)$: $P'(q) \cdot q + P(q) - C'(q) = 0$

which yields:

$$MR(q^m) = P'(q) \cdot q + P(q) = C'(q)$$
Properties of Monopoly price

1) Monopoly price is a non decreasing function of marginal cost.
Consider two alternative cost functions for a monopolist $C_1(q), \ C_2(q)$
Assume differentiability and $C'_2(q) > C'_1(q)$ for $q > 0$.
If $C'_1(q)$

$$p_1 q_1^m - C_1(q_1^m) \geq p_2^m q_2^m - C_1(q_2^m)$$

If $C'_2(q)$

$$p_2^m q_2^m - C_2(q_2^m) \geq p_1 q_1^m - C_2(q_1^m)$$

hence

$$[C_2(q_1^m) - C_2(q_2^m)] - [C_1(q_1^m) - C_1(q_2^m)] \geq 0$$
Properties of Monopoly price

Or since differentiable:

\[ \int_{q_2^m}^{q_1^m} \left[ C'_2(x) - C'_1(x) \right] dx \geq 0 \]

Since by assumption \( C'_2(x) > C'_1(x) \) for all \( x \), hence \( q_1^m \geq q_2^m \)

Monopoly price is a **non decreasing** function of marginal cost
Properties of Monopoly price

- 2) the elasticity $\varepsilon$ is greater than 1 at $p^m$.

In fact:

$$\frac{p^m - c'(D(p^m))}{p^m} = \frac{1}{\varepsilon} = \frac{-D(p^m)}{p^m} \cdot \frac{1}{D'(p^m)}$$

And

$$\varepsilon = \frac{p^m}{p^m - c'(D(p^m))} > 1$$
Monopoly theory – the Deadweight loss
Oligopoly theory

• A refresher on Game Theory
• Bertrand Competition
• Cournot Competition
• Bertrand with capacity constraints
• Application: merger in Cournot
Before going into models let’s have a brief review of Game Theory.

Consider two firms i=1,2
with $\pi^i(a_i, a_j)$ twice continuous and differentiable, $a_i \in \mathbb{R}$

**Nash Equilibrium:**

$\pi^i(a^*_i, a^*_j) \geq \pi^i(a_i, a^*_j)$, for every $a_i, i=1,2$.

f.o.c. $\pi^i_i(a^*_i, a^*_j) = 0$

s.o.c. $\pi^i_{ii}(a^*_i, a^*_j) \leq 0$

Assume strict concavity $\pi^i_{ii}(a^*_i, a^*_j) < 0$ for every $(a_i, a_j)$ then f.o.c. is sufficient for Nash Equilibrium.

Let’s define the reaction function $R_i(a_j)$ s.t.:

$$\pi^i(R_i(a^*_j), a^*_j) = 0 \quad (1)$$

$a_i = R_i(a_j)$ is unique by strict concavity.
Another way to define the N.E.:
\((a^*_i, a^*_j)\) s.t. \(a^*_i = R_i(a^*_j)\)

Hence by differentiating (1)
\[\pi_{ii}^i(R_i(a_j), a_j) \cdot R_i'(a_j) + \pi_{ij}^i(a_i, a_j) = 0\]

i.e.

\[R_i'(a_j) = \frac{\pi_{ij}^i(R_i(a_j), (a_j))}{-\pi_{ii}^i(R_i(a_j), (a_j))}\]

Therefore sign \((R_i'(a_j)) = \text{sign} (\pi_{ij}^i(a_i, a_j))\)
Game Theory – a refresher

- A useful concept at this stage is distinguish between strategic complements and strategic substitutes.
- We can determine it by looking to the sign of the derivative of the Reaction Function.
- These two concepts are used for comparative statics.
Oligopoly theory - Bertrand

Short run price competition

- Consider two firms producing identical goods with constant marginal cost, c.

- $D(p)$, total demand.

- Demand for firm $i$ is $d_i(p_i,p_j)$

$$d_i(p_i,p_j) = \begin{cases} D_i(p_i) & \text{if } p_i < p_j \\ \frac{1}{2} D_i(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

There is a unique equilibrium $p_i=p_j=c$ $\rightarrow$ Bertrand Paradox
Oligopoly theory - Cournot duopoly

• This is a one-stage game in which firms choose their quantities simultaneously.

• Profit function for firm i can be generally defined as

\[ \pi_i(q_i, q_{-i}) = q_i \cdot P(q_i + q_{-i}) - C_i(q_i) \]

• Each firm maximizes its profit given the quantity chosen by the other firm.

• Assuming that the profit function \( \pi_i \) is strictly concave in \( q_i \) and twice differentiable, we can maximize firms’ profit:

\[
\text{Max}_{q_i} \quad \pi_i(q_i, q_{-i}) = q_i \cdot P(q_i + q_{-i}) - C_i(q_i)
\]

f.o.c. \[ P(q_i + q_{-i}) - C'_i(q_i) + q_i P'(q_i + q_{-i}) = 0 \]

This FOC implicitly defines \( q_i = R_i(q^*_{-i}) \), where \( R_i \) is firm i’s reaction curve:

\[ \pi^i_i (R_i(q^*_{-i}), q^*_{-i}) = 0. \]

• Assumption: firm i’s marginal profit is decreasing in the other firms’ quantity. This implies that reaction function are downward sloping (strategic substitutes).
Oligopoly theory – Cournot duopoly

• Dealing with Monopoly we defined the Lerner Index (price-cost margin). \( L_i = \frac{P - C'_i}{P} \)

• Now, let \( \alpha_i \) be the firm i’s market share: \( \alpha_i = \frac{q_i}{Q} \)

• And the inverse elasticity of demand: \( \frac{1}{\varepsilon} = -\frac{P'}{PQ} \)

• In the Cournot model, the Lerner Index is equal to:

\[
L_i = \frac{P - C'_i}{P} = \frac{-q_i}{P} P'(Q) = \frac{-q_i}{P} \frac{Q}{Q} P'(Q) = \frac{-q_i}{Q} \frac{P'(Q)}{P} Q = \frac{\alpha_i}{\varepsilon}
\]

• Furthermore we can define the index of concentration, known as Herfindhal Index:

\[
H \equiv \sum_{i=1}^{n} \alpha_i^2
\]
Oligopoly theory - Cournot
Oligopoly theory – Cournot, a duopoly example

Quantity competition

- Consider two firms producing identical goods

- Demand \( D(p) = 1 - p \)

- \( C_i(q_i) = c_i \cdot q_i \)

- \( P(q_i, q_j) = 1 - q_i - q_j \)

Individual firm optimization, taking action by other firm as given

Max \( q_i \), \( \pi_i(q_i, q_j) = q_i (1 - q_i - q_j) - c_i q_i \)

f.o.c. \( q_i \) \(-q_i + 1 - q_i - q_j - c_i = 0\) \( \rightarrow \) \( q_i = (1 - q_j - c_i)/2 \)

s.o.c. \( q_i \) \(-2 < 0\)
Oligopoly theory – Cournot, a duopoly example

Hence we can derive the Best Response for Firm 1 and 2

\[ q_1 = \frac{1-q_2-c_1}{2} = BR_1(q_2) \quad \quad q_2 = \frac{1-q_1-c_2}{2} = BR_2(q_1) \]

And the Cournot equilibrium:

\[ q_i = \frac{1-2c_i + c_j}{3} \]

Finally the Lerner index: \( L_i = \frac{1-q_i-q_j-c_i}{1-q_i-q_j} \) and the Herfindhal: \( H = \frac{q_i^2 + q_j^2}{Q^2} \)

Hence for the duopoly case: \( Q = q_1 + q_2 = \frac{1-2c_1 + c_2}{3} + \frac{1-2c_2 + c_1}{3} = \frac{2-c_1-c_2}{3} \)

It follows that in the case in which the firms are identical the total quantity produced is

\[ Q = \frac{2}{3} (1-c) \]
Oligopoly theory - Cournot (n firms)

- n identical firms with cost function \( C(q_i) = c \cdot q_i, \ i=1,\ldots,n \)
- Market Demand: \( P(Q)=1-Q, \ Q=\Sigma_{i=1,\ldots,n} q_i \)
- Profit: \( \pi_i(q_1,\ldots, q_n)= P(\Sigma_{i=1,\ldots,n} q_i) q_i - C(q_i) \)

**Individual profit maximization**

f.o.c. \( q_i \)  
\[
P'(Q) + P'(Q)q_i = c
\]

Or equivalently \( 1-Q- q_i = c \)

Hence individual quantity produced is \( q_i = 1-Q- c \)

Summing over \( i, Q=n(1-Q- c) \).

**Equilibrium:**

\[
Q^* = \frac{n}{n+1} (1-c)
\]

\[
p^* = \frac{1 + nc^*}{n+1}
\]

\[
q^* = \frac{1-c}{n+1}
\]

\[
\pi = \left( \frac{1-c}{n+1} \right)^2
\]
Oligopoly theory - Cournot

We can also find the relation between the two indices.

In fact the average weighted Lerner index is equal to the Herfindahl index divided by the price elasticity of demand, i.e. \( \Sigma_{i} \alpha_{i} L_{i} = H/ \varepsilon. \)

In a monopoly, \( H = 1. \)

It can also be computed to hold for a general \( n \)-firm Cournot oligopoly (but requires a bit of calculus).

In fact \( q^{*} = \frac{1-c}{n+1} \) and \( Q^{*} = \frac{n}{n+1}(1-c) \)

Then \( \alpha_{i} = \frac{q_{i}^{*}}{Q^{*}} = \frac{1}{n} \).

Hence \( H \) tends to zero as \( n \) increases, since is the sum of the squares of market shares.
Oligopoly theory – Bertrand with capacity constraints

Two stage game.

First stage: Each firm simultaneously chooses its capacity.

Second stage: Each firm simultaneously chooses its price.

Efficient Rationing Rule.
Suppose \( p_1 < p_2 \) and \( q_1 \) capacity constraint of Firm 1 with \( q_1 < D(p_1) \)

Residual demand for Firm 2 is

\[
D(p_2) = \begin{cases} 
D(p_2) - q_1 & \text{if } D(p_2) > q_1 \\
0 & \text{otherwise}
\end{cases}
\]

This rationing is called efficient since it maximizes consumer surplus
Firm 2 faces translated a demand curve
Oligopoly theory – Bertrand with capacity constraints

Model assumptions

• $Q = D(p) = 1 - p \iff p = 1 - q_1 - q_2$

• In stage 1, firms choose capacity simultaneously $q_i$
  
  – Unit cost of capacity acquisition $c_0 \in [\frac{3}{4}, 1]$

• In stage 2, firms simultaneously choose prices
  
  – Mc of production
    
    $$
    c = \begin{cases} 
    0 & \text{if } q_i \leq q_i \\
    \infty & \text{if } q_i > q_i
    \end{cases}
    $$

  – We apply the efficient rationing rule

Solution – first focus on second stage and solve for the equilibrium in the second stage game given the capacity choices in the first stage.

We can restrict our attention to $q_i, q_j \in [0, \frac{1}{3}]$

Claim: $p^* = 1 - q_1 - q_2$ is the unique equilibrium

Let’s explore the two cases: 1) if $p_i < p^*$
  
  2) if $p_i > p^*$
Oligopoly theory – Bertrand with capacity constraints

Stage 1:
From the solution we know that the profit of firms takes the form of the Cournot profit

$$\pi_i = q_i (1 - q_j - q_i)$$ and we can proceed with the usual maximization.

From the Cournot solution we've derived earlier we get $$q_i = q_i = (1 - q_j)/2$$

This model shows that in this case everything is as if the two firms put outputs equal to their capacities on market and an auctioneer equalled supply and demand. The difference is that firms choose the market-clearing price themselves.

Capacity constraint softens price competition.

Check also Kreps and Scheinkman (1983)
Application to merger in Cournot

Cournot case
- At least 3 identical firms with mc=c<1
- Linear demand Q=1-p

\[
\begin{aligned}
Q &= \sum_{j=1}^{n} q_j \\
Q_i &= Q - q_i
\end{aligned}
\]

Before merger

Max \( q_i \pi_i = q_i (1 - Q_i + q_i - c) \)

f.o.c. \( q_j \)

\( q_j = (1 - Q_i - c)/2 \)

By symmetry \( q_i = q \) for all \( i \)

Hence \( q = [1-(n-1) q - c]/2 \)

That yields

\[
q = \frac{1 - c}{n + 1}
\]

\[
\pi_i = \left( \frac{1 - c}{n + 1} \right)^2
\]
Application to merger in Cournot

**Merger of two firms**

- Suppose that F1 and F2 merge given \((q_3, \ldots, q_n)\)
  
  \[
  \max_{q_1, q_2} \pi^1 + \pi^2 = q_1 (1 - Q - c) + q_2 (1 - Q - c) = (q_1 + q_2) (1 - q_1 - q_2 - q_3 - \ldots - q_n)
  \]

Let \(q_1 + q_2 = q_{1+2}\)

Then \(\max q_{1+2} \cdot (1 - q_{1+2} - q_3 - \ldots - q_n - c)\)

Merger of two firms it’s like reducing the market of one firm.

The **merger paradox**: with constant marginal cost merger is less profitable.

**Comparison**

1) Profit of the two firms

\[
2 \left( \frac{1-c}{n+1} \right)^2 > \left( \frac{1-c}{n} \right)^2
\]

2) Profit of the other firms

\[
\left( \frac{1-c}{n+1} \right)^2 < \left( \frac{1-c}{n} \right)^2
\]

3) Industry profits

\[
n \left( \frac{1-c}{n+1} \right)^2 < (n-1) \left( \frac{1-c}{n} \right)^2
\]
Oligopoly theory – Final Remarks

• Strategic interactions are key to understanding oligopolistic markets

• In Bertrand we obtain a perfect competitive outcome.

• We can move away from the competitive outcome with the Cournot model or with the introduction of capacity constraints.

• There is another possibility: the possibility of collude and make a cartel and maximize the aggregate profits.

• Hence we must analyze collusion and incentives to do it.
Collusion – Roadmap

• Collusion: economic perspective
• Model
• Basic results and Market concentration
• Information lag
• Fluctuating demand
• Green and Porter (1984)
Collusion – Economic Perspective

• Collective exercise of market power, i.e. a situation where firms raise price beyond what would be consistent with short term profit incentives.

• Hence, firms have an incentive to take advantage of their competitors in the short term.

• Default may be avoided if it triggers a period of intense competition during which the benefits of the collusive outcome are lost (retaliation).

• It is conceivable that collusion could arise from market interactions without concertation.

• Explicit “agreements” or “concertation” cannot be enforced by law because they are illegal

• Hence, only self interest can support collusive outcomes, irrespective of whether there is concertation or not.

• No fundamental distinction between cartels and “tacit” collusion.

• But “concertation” may help in supporting collusive outcomes.
Collusion – The model

- Two firms producing perfect substitutes with the same constant marginal cost, c.
- Bertrand competition for T period (finite/infinite)
  - \( \sum_{t=0}^{T} \delta^t \pi^i(p_{it}, p_{jt}) \) is the present discounted value of F_i’s profit
  - \( \delta \) = discount factor
  - \( H_t=(p_{10}, p_{20}, \ldots, p_{1t-1}, p_{2t-1}) \) history at time t
  - Perfect equilibrium: for any history \( H_t \) at date t, F_i’s strategy from date t on maximizes the present discounted value of profits given F_j’s strategy from date t on.

- T finite, finite horizon (Bertrand solved T times):
  - \( t=T \) equilibrium \( p_{iT}=c \) for every i
  - \( t=T-1 \) equilibrium \( p_{iT-1}=c \) for every I
  - .....  
  - From now on T is infinite.
Collusion – Basic Results and Market concentration

Monopoly outcome

- Let \( p_m \) the monopoly price
- Trigger strategy:
  \[
  \begin{cases}
  at t = 0 \ F, c h a r g e s p_m \\
  f r o m t > 0
  \\
  p_{it} = p_m \ i f \ H_t = (p^*, p_1^*, ..., p_m) \\
  p_{it} = c \ f o r e v e r
  \end{cases}
  \]
- Payoff of the following strategy
  \[
  \frac{\pi_m}{2} (1 + \delta + \delta^2 \ldots) = \frac{1}{1-\delta} \frac{\pi_m}{2} \geq \pi_m
  \]
- Payoff upon the best deviation \( \pi_m + 0 + 0 + \ldots + 0 \).

- \([IC]\) \( (1-\delta)^{-1} (\pi_m/2) \geq \pi_m \iff \delta \geq \frac{1}{2} \)

Folk Theorem

Any pair of profits \( (\pi_1, \pi_2) \) s.t. \( \pi_1 > 0, \pi_2 > 0, \pi_1 + \pi_2 \leq \pi_m \) there exists a threshold \( \delta^* \) (large enough) such that these pair of profits can be sustained as part of a collusive equilibrium.

For \( n > 2 \) collusion harder

\([IC]\) \( (1-\delta)^{-1} (\pi_m/n) \geq \pi_m \)
Collusion – Information lag

A firm deviation is detected by its rival only two periods after

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If \( n=2 \)

\[ [IC] \ (1-\delta)^{-1} \left( \frac{\pi_m}{2} \right) \geq \pi_m + \delta \pi_m \]
Collusion – Observed fluctuating Demand

A firm decide to collude or not depending from the trend (boom, recession…). During booms they deviate more.

- Demand iid over time with \( D_2(p) > D_1(p) \)

\[
\frac{1}{2} \rightarrow low \ q = D_1(p) \quad \frac{1}{2} \rightarrow high \ q = D_2(p)
\]

- At each period, each firm learns the current state of the demand before choosing price.

- We look for \((p_1, p_2)\) s.t.
  - Each firm charges \( p_s \) when \( s = 1, 2 \)
  - \((p_1, p_2)\) is an equilibrium
  - The expected profit of each firm along the equilibrium path is not Pareto dominated by other equilibrium payoffs.

- Let \( V = \text{expected present profit along the equilibrium path} \)

\[
V = \sum_{t=0}^{\infty} \delta^t \left( \frac{1}{2} \frac{D_1(p_1)}{2} (p_1 - c) + \frac{1}{2} \frac{D_2(p_2)}{2} (p_2 - c) \right) =
\]

\[
\left( \frac{1}{4} D_1(p_1)(p_1 - c) + \frac{1}{4} D_2(p_2)(p_2 - c) \right) \frac{1}{1 - \delta}
\]
Collusion – Fluctuating Demand

Trigger strategy: if deviation each firm will charge \( p=c \) forever.

**Case 1**

Full collusion outcome: they charge monopoly price with Demand \( s : p_s^m, \pi_s^m \)

Then

\[
V = \frac{\pi_1^m + \pi_2^m}{4(1 - \delta)}
\]

\([\text{IC}_s]\) \quad \frac{1}{2} \cdot \pi_s^m + \delta V \geq \pi_s^m + 0 \text{ with state } s=1,2.

\(\text{IC}_2\) is more difficult to satisfy than \(\text{IC}_1\)

\([\text{IC}_s]\) \quad \frac{1}{2} \cdot \pi_s^m + \delta V \geq \pi_s^m \text{ iff } \delta V \geq \frac{1}{2} \cdot \pi_s^m

While \(\text{IC}_2 \delta \geq \delta_0 = \frac{2 \pi_2^m}{(\pi_1^m + 3 \pi_2^m)}\)

Where \(\frac{3}{5} > \delta_0 > \frac{1}{2}\)
Collusion – Observed fluctuating Demand

Case 2: Can we sustain some level of collusion when $\delta < \delta_0$, where $\delta \in \left[ \frac{1}{2}, \delta_0 \right]$

\[
\max_{p_1, p_2} \frac{1}{2} \pi_1 \left( p_1 \right) + \frac{1}{2} \pi_2 \left( p_2 \right) = V
\]

s.t. \hspace{1cm} IC_1

\[IC_2 \pi_2 \left( p_2 \right) + \delta V \geq \pi_2 \left( p_2 \right) \hspace{1cm} \text{iff} \hspace{1cm} k \pi_1 \left( p_1 \right) \geq \pi_2 \left( p_2 \right), \text{ with } k = \frac{\delta}{2-3\delta} \]

\[
\max_{p_1, p_2} \pi_1 \left( p_1 \right) + \pi_2 \left( p_2 \right)
\]

s.t. \hspace{1cm} IC_1 \hspace{1cm} k \pi_2 \left( p_2 \right) \geq \pi_1 \left( p_1 \right)

IC_2 \hspace{1cm} k \pi_1 \left( p_1 \right) \geq \pi_2 \left( p_2 \right)

IC_2 \text{ is binding. With low demand they charge monopoly price. With high demand price falls below monopoly price. We may observe countercyclical price movements.}
Collusion - Unobserved demand

• Let’s consider a simplified version of Green-Porter model.

• Two firms choose price every period

• The goods are perfect substitutes and are produced at constant marginal cost, $c$, so consumer all buy from the low-price firm.

• Demand is split in halves if both firms charge the same price.

• Two possible state of nature:
  - With probability $\alpha$ there is no demand for the product sold by the duopolist
  - With probability $1-\alpha$ there is a positive demand $D(p)$

• Realization of demand are independently and identical distributed.

• A firm that does not sell at some date is unable to observe whether the absence of demand is due to the realization of the low demand state or to its rival’s lower price.
Collusion - Unobserved demand

• It’s always common knowledge that at least firm observes zero sales:
  – Either If there is absence of demand no firm makes profits.
  – Either If one firm undercuts price, it knows that the other is going to make no profits.

• As usual there are collusion phase and punishment phase (that will last T periods).

• Punishment phase will last T periods because zero sales could be triggered by the absence of demand and not by a deviation.

• In a non repeated game or finitely repeated game both firm charge the competitive price.

• Let \( V^+ \) (respectively \( V^- \)) be the present discounted value of a firm’s profit from date \( t \) on, assuming that at date \( t \) the game is in the collusive phase (respectively, starts the punishment phase)

• By stationarity \( V^+ \) and \( V^- \) do not depend on time, then:
  – \( V^+ = (1-\alpha)(\pi_m/2+\delta V^+)+\alpha(\delta V^-) \)
  – \( V^- = \delta^T V^+ \)
Collusion - Unobserved demand

- The IC in this case are $V^+ \geq (1-\alpha)(\pi^m/2+\delta V^-)+\alpha(\delta V^-)$ that states that no firm will undercut price.

- Here is the tradeoff for each firm since if undercuts price it gets $\pi^m > \pi^m/2$, hence we need to deter undercutting that $V^-$ will be sufficiently lower than $V^+$.

- Plugging the IC into the present discounted value of collusive phase we get: $\delta(V^+ - V^-) \geq \pi^m/2$.

- By rearranging the present discounted value of both phases we get:
  \begin{align*}
  V^+ &= \frac{(1-\alpha)\pi^m/2}{1-(1-\alpha)\delta - \alpha \delta^{T+1}} \\
  V^- &= \frac{(1-\alpha)\delta^T\pi^m/2}{1-(1-\alpha)\delta - \alpha \delta^{T+1}}
  \end{align*}

- Finally substituting it into $\delta(V^+ - V^-) \geq \pi^m/2$, we get $1 \leq 2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1}$.

- Hence starting in the colluding phase the firm must solve:
  \[\max V^+ \text{ s.t. } 1 \leq 2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1}\]

- To maximize profits we need to choose the smallest $T$ that satisfies the incentive constraints, since longer punishment phase reduce the expected profit.
Multimarket contact and collusion

- Bernheim and Whinston (1990) identify a number of circumstances, typically implying asymmetries between firms or markets, in which multimarket contact facilitates collusion by optimizing the allocation of available enforcing power between markets with a price competition model.

- They show that when firms and markets are identical and there are constant returns to scale, multimarket contact does not strengthen firms' ability to collude (irrelevance result).

- In multimarket case a deviation get punished in several markets, but the firm can also deviate in all markets at the same time by pooling of the IC across markets.

- Three main conclusions:
  - Multimarket contact can have real effects; in a wide range of circumstances, it relaxes the incentive constraints that limit the extent of collusion.
  
  - Firms' gain from multimarket contact by behaving in ways that have been noted in previous empirical discussions of multimarket firms. This suggests that multimarket contact may indeed have effects in practice.
  
  - Even when multimarket contact does have real effects, these effects are not necessarily socially undesirable.