Multiproduct firms, nonlinear pricing and price discrimination
Lecture notes

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1 Introduction

In many industries, firms produce several products. This raises a new set of positive (pricing, product positioning, economies of scope) and normative (welfare distortion, mergers, unbundling) questions.

We will distinguish between environments where consumers consume a single unit of a product and environments when they consume several products and/or several units of the same product.

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2 Unit demand consumers

Even when firms are multiproduct, as long as consumers only buy one unit, the estimation of demand can proceed as before. What changes is firm behavior.

2.1 Pricing

Suppose monopolist produces two products with demand $D_j(p_1, p_2)$, $j = 1, 2$. Each product is produced under a separate constant marginal cost technology. Optimal prices solve

$$\max_{p_1, p_2} (p_1 - c_1)D_1(p_1, p_2) + (p_2 - c_2)D_2(p_1, p_2)$$
The FOCs can be expressed in matrix notations as

\[
\begin{bmatrix}
p_1 - c_1 \\
p_2 - c_2
\end{bmatrix} = \left[ \begin{array}{cc}
\frac{\partial D_1}{\partial p_1} & \frac{\partial D_2}{\partial p_1} \\
\frac{\partial D_1}{\partial p_2} & \frac{\partial D_2}{\partial p_2}
\end{array} \right]^{-1} \begin{bmatrix}
D_1 \\
D_2
\end{bmatrix}
\]

Let \( D_{ij} = \frac{\partial D_i}{\partial p_j} \). Solving for \( p_1 - c_1 \), we get:

\[
p_1 - c_1 = -\frac{D_1 D_{22} - D_2 D_{21}}{D_{11} D_{22} - D_{12} D_{21}} = -\frac{D_1}{D_{11} - \frac{D_{12} D_{21}}{D_{22}}} + \frac{D_2 D_{21}}{D_{11} D_{22} - D_{12} D_{21}}
\]

Prices are higher when \( D_{12} > 0 \) (products are substitutes - which comes directly from demands based on the discrete choice models we have seen so far). They depend on both own and cross price elasticities.

**Appl’n: Leslie (2005)**

Leslie asks whether a Broadway theater sets prices for its different seat categories optimally for a specific play "Seven Guitars". Setting corresponds to multiproduct monopolist facing unit-demand consumers.
Demand specification distinguishes between regular tickets, tickets bought with a coupon and tickets bought on the same day at a booth. The outside good is all other Broadway shows. Utilities from each alternative is given by:

\[
U_{ijt} = \begin{cases} 
\xi_{it}q_j(\delta_1 y_i^\delta_2 - p_{jt})^\eta & j = h, m, l \\
\xi_{it}q_h(\delta_1 y_i^\delta_2 - p_{ct})^\eta & \text{coupons} \\
\xi_{it}q_h((\delta_1 y_i^\delta_2 - p_{bt} - \tau_1 y_i - \tau_2)^\eta & \text{tickets bought at booth} \\
(\delta_1 y_i^\delta_2 - p_o)^\eta & \text{outside good}
\end{cases}
\]

where \(y_i\) is the income of individual \(i\) (\(\delta_1 y_i^\delta_2\) is the budget for entertainment), \(\xi_{it}\) is individual \(i\)’s taste for the show at time \(t\) (assumed to be distributed according to an exponential \(\exp(X_t/\beta)\)). The model also includes a probability of receiving a coupon in any period (if not, then alternative \(c\) is not available), and the fact that some categories may fill up and thus are not available when consumers asks for it.

Conditional on a value for parameters, on a sequence over consumers and on draws over coupons, we can easily partition the set of \((y_i, \xi_{it})\) according to which alternative is chosen and thus predict market shares (the distribution of \(\xi_{it}\) is assumed
and the distribution of $y_i$ is known) → Simulated MLE, result is estimated demand $q_{jt}(p_t, X_t, \hat{\theta})$

**Counterfactual:** $\max \{p_{ht}, p_{mt}, p_{lt}, p_{bt}, p_{ct}\} \sum_t \sum_{j \neq o} p_{jt}q_{jt}(p_t, \cdot)$. Explores several scenarios

**TABLE 5: Results of Counterfactual Experiments**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Revenue ($\text{million}$)</th>
<th>Utility</th>
<th>Ave. Attendance</th>
<th>$p_t$</th>
<th>$p_m$</th>
<th>$p_b$</th>
<th>$p_b$</th>
<th>$p_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>4.6951</td>
<td>NA</td>
<td>661.56</td>
<td>16.93</td>
<td>29.20</td>
<td>55.08</td>
<td>27.53</td>
<td>31.01</td>
</tr>
<tr>
<td>Base-A</td>
<td>6.2698</td>
<td>3.5859</td>
<td>906.86</td>
<td>16.93</td>
<td>29.20</td>
<td>55.08</td>
<td>27.53</td>
<td>31.01</td>
</tr>
<tr>
<td>Base-B</td>
<td>7.8965</td>
<td>3.5775</td>
<td>864.11</td>
<td>23.90</td>
<td>29.80</td>
<td>60.22</td>
<td>30.11</td>
<td>45.26</td>
</tr>
<tr>
<td>Uniform</td>
<td>8.0204</td>
<td>3.6039</td>
<td>809.57</td>
<td>50.04</td>
<td>50.04</td>
<td>50.04</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>No-Booth-A</td>
<td>6.7301</td>
<td>3.5837</td>
<td>873.01</td>
<td>16.93</td>
<td>29.20</td>
<td>55.08</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>No-Booth-B</td>
<td>8.3495</td>
<td>3.5925</td>
<td>873.73</td>
<td>22.28</td>
<td>38.33</td>
<td>51.53</td>
<td>NA</td>
<td>43.23</td>
</tr>
<tr>
<td>Booth not 50%</td>
<td>8.4516</td>
<td>3.5900</td>
<td>850.30</td>
<td>24.47</td>
<td>40.86</td>
<td>54.21</td>
<td>38.05</td>
<td>46.32</td>
</tr>
<tr>
<td>Non-sticky</td>
<td>8.0194</td>
<td>3.5800</td>
<td>887.37</td>
<td>24.11</td>
<td>30.11</td>
<td>59.73</td>
<td>29.87</td>
<td>46.03</td>
</tr>
</tbody>
</table>
2.2 Product positioning

So far, we have considered how the monopolist should optimize its pricing structure, given the set of products (with fixed attributes) that it sells. We now endogenize the products. Suppose the monopolist faces two types of consumers, characterized by their taste for quality (quantity), \( \theta \in \{ \theta_L, \theta_H \}, \) \( 0 < \theta_L < \theta_H \) (the proportion of high types is \( \lambda \)). Types are private information. Monopolist faces a cost of \( \frac{1}{2}q^2 \) for producing at quality \( q \). Consumers get utility \( \theta q - p \) from consuming a good of quality \( q \) and paying \( p \) for it.

**First best benchmark**: Qualities served to each type solves \( \max_q \theta q - \frac{1}{2}q^2 \) for \( \theta \in \{ \theta_L, \theta_H \} : q_{LB}^{FB} = \theta_L, \ q_{HB}^{FB} = \theta_H. \) First best can be achieved when types are known to all through *first degree (perfect) price discrimination*. Monopolist sets prices such that \( p_i = \theta_i q_i^{FB}. \)
In practice types are not observed so the monopolist solves (conditional on "full coverage")

$$\max_{q_L, q_H, p_L, p_H} \lambda(p_H - \frac{1}{2}q_H^2) + (1 - \lambda)(p_L - \frac{1}{2}q_L^2)$$

subject to $\theta_H q_H - p_H \geq \theta_H q_L - p_L$

and $\theta_L q_L - p_L \geq 0$

This is the standard second degree price discrimination problem (screening problem). The solution is

$q_H^* = \theta_H, \ q_L^* = \theta_L - \frac{\lambda}{1-\lambda} \Delta \theta$ (no distortion at the top, downward distortion elsewhere)

$\Delta p = \theta_H \Delta q > \Delta c = \Delta q \frac{q_H^* + q_L^*}{2}$ (price differences are not aligned with marginal costs differences)

Total welfare is lower but profits are higher.
**Extension 1:** Types are continuously distributed according to a cdf $F$ (and quality can take continuous values too)

Suppose preferences take the form $u(q, \theta) - P(q)$. Given any price function $P(q)$, $q(\theta) = \arg\max_q u(q, \theta) - P(q)$, yielding a utility level $v(\theta) = \max_q u(q, \theta) - P(q)$. The monopolist solves

$$\max_{q(\cdot)} \int (P(q(\theta)) - C(q(\theta))) \, dF(\theta) = \int (u(q(\theta), \theta) - C(q(\theta) - v(\theta))) \, dF(\theta)$$

under the usual incentive compatibility and individual rationality constraints which can be shown to be equivalent to $q(\cdot)$ monotonically increasing, $v'(\theta) = u_\theta(q(\theta), \theta) > 0$ and $v(\theta) \geq 0$. After integration by parts (substituting $v'$) we get

$$\max_{q(\cdot)} \int \left( u(q(\theta), \theta) - C(q(\theta) - \frac{1 - F(\theta)}{f(\theta)} u_\theta(q(\theta), \theta)) \right) \, dF(\theta)$$

This expression can be maximized point-wise, yielding as FOC

$$u_q(q(\theta), \theta) = C_q(q(\theta) + \frac{1 - F(\theta)}{f(\theta)} u_{\theta q}(q(\theta), \theta))$$
Under a single crossing condition $u_{\theta q} > 0$ the second order condition $q(.)$ increasing is satisfied. We have as before no distortion at the top and downward distortion in the quality elsewhere.

**Extension 2:** Types are multi-dimensional

Issues now are whether monopolist has enough instruments and to what extent those multiple dimensions can be collapsed into a "sufficient statistics". Also which ones are observable and thus amenable to "personalized pricing" (third degree price discrimination) and which ones aren’t. See Rochet and Stole (2002) as an example.

**Extension 3:** Market is an oligopoly

Consider again the model above with several firms and consumer preferences given by $u^j(q_j, \theta) - P^j(q_j), j = 1, ..., n$. Given $P^j(.)$, the indirect utility of consumer $\theta$ from firm $j$’s offering is given by $v_j(\theta) = \max_{q_j} u^j(q_j, \theta) - P^j(q_j)$. Thus from firm $j$’s perspective, consumer $\theta$’s best alternative is

$$v_j(\theta) = \max \{0, v_1(\theta), ..., v_{j-1}(\theta), v_{j+1}(\theta), ...\}$$
The best response by firm \( j \) is the same as the best response by a monopolist which faces a consumer with utility \( u^j(q, \theta) - P^j(q) \) and an outside option \( v^j(\theta) \). The fact that the outside option depends on types implies that the identity of marginal consumer is harder to pin down ex-ante. Extra structure is needed in general (Stole, 2005).

**Horizontal differentiation** (Spulber, 1989) Suppose for example the source of heterogeneity among consumers is location. Let \( \theta \) measure the distance of a consumer to firm 1 located at 0 on the hotelling line. Assume further that \( u^1_{\theta}(q, \theta) < 0 < u^2_{\theta}(q, \theta) \) and that \( u^1_{q\theta}(q, \theta) < 0 < u^2_{q\theta}(q, \theta) \). Consider firm 2’s problem, taking \( P_1(q) \) as fixed. \( v_2(\theta) = \max \{0, \max_q u^1(q, \theta) - P_1(q)\} \). By the envelope thm, \( v_2'(\theta) = u^1_{\theta}(q(\theta), \theta) < 0 \). Let \( \theta^* \) be the marginal consumer (the consumer indifferent between the offer from firm 1 and the offer from firm 2). Firm 1’s market is \([0, \theta^*]\) and firm 2’s market is \([\theta^*, 1]\). Thus we can separately determine the allocation of quality and market shares. Price competition is entirely focussed on the marginal consumer (each firm acts as a second-degree price discriminating monopolist on its market when it determines \( q(\theta) \)). Here competition lower prices but does not affect quality distortion - relative to the case where both firms would be
under single ownership.

**Vertical heterogeneity.** Issue now is to introduce some "friction" to avoid perfect Bertrand competition. Possibilities include cost asymmetries (Stole, 1995), precommitment to qualities ex-ante (Champsaur and Rochet, 1989), or private information about costs (Biglaiser and Mezzetti, 2000).
Application: McManus (2007)

Looks at evidence of second degree price discrimination and quality distortion in the specialty coffee market around the University of Virginia.

Demand specification allows for a vertical dimension of preference for products and an horizontal dimension of preference due to location. In each period $t$ (portion of the day), each consumer $i$ located at location $l$ makes a purchase ($j$ is product, $x$ is product line (drip, regular, specialty)):

$$U_{ijlt} = \begin{cases} \beta_{xi} q_j x + \alpha p_j + \delta D_{jl} + \xi_j + \epsilon_{ijlt} \\ X_t \phi + \epsilon_{iolt} \end{cases} \quad \text{for } j \in J_t$$

outside good ($X_t$ contains period dummies)

where $q_j$ is the size, $D_{jl}$ is distance from consumer’s location and $\xi_j$ is unobserved quality. Assuming a lognormal distribution for $\beta_{x_i}$ and a logit distribution for $\epsilon$, and given the known distribution of locations, one can then compute expected market shares for each product during period $t$ as a function of the value of parameters (also knows total population) (SMLE)
Combines demand estimates with costs information to check several predictions of the theory:

"No distortion at the top": at the largest size, the marginal benefit of an extra ounce should be equal to the extra cost. Marginal benefit of increase the size of product $j$ by one ounce is computed as $MB_j(\theta) = \sum_l \sum_t \int_{\beta_{xi}} \gamma_{x_i} \beta_{xi} q_j^{(\gamma x - 1)} \frac{\gamma_{x_i}}{\alpha_i}$. Marginal cost is computed on the basis of cost data.

Role of outside good, and substitution across product categories.
Explains difference between expressos and specialty coffee by the fact that expressos are lower margin substitutes for specialty coffees, and low and flat distortion for drip coffee by fact that competes most fiercely with outside good.
2.3 Economies of scope

When firms produce several products a natural question that arises concern the existence and importance of economies of scope. Let $Y \in R^2$ a vector of outputs (two products). Let $P_1Y$ the projection of $Y$ on the $x$–axis and $P_2Y$ the projection of $Y$ on the $y$–axis. Let $C(Y)$ be the cost to produce output $Y$.

**Definition 1:** There are economies of scope in region $A$ if $C(Y) \leq C(P_1Y) + C(P_2Y)$ for all $Y \in A$ such that $P_1Y, P_2Y \in A$

This is especially relevant in the context of mergers (synergy and thus efficiency motivation for mergers versus market power motivation) and in the context of regulation (unbundling, dismantling of regulated monopolies).
Non-identification result

In the NEIO, we have the FOC condition for optimal prices to infer firms’ marginal costs. A common assumption of the pricing model is that firms have constant marginal costs. Can we identify else than marginal costs (under the assumption that products are technology independent and marginal costs are constant)? Consider the profit function of a firm with cost function $C(D_1(p_1, p_2), D_2(p_1, p_2))$:

For simplicity, suppose the firm produces two goods, and rewrite its optimization problem as

$$\max_{p_1, p_2} p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2) - C(D_1(p_1, p_2), D_2(p_1, p_2))$$

For given observed optimal prices, the FOCs yields two nonlinear equations where $C_{q_1}(D_1(p_1, p_2), D_2(p_1, p_2))$ and $C_{q_2}(D_1(p_1, p_2), D_2(p_1, p_2))$. This is too little to infer the cost function unless costs are indeed independent across products and marginal costs are constant. The issue here is that we cannot in general infer information of greater dimension than the information we observe. Additional sources of variations (observed prices under different demand conditions) can help identify
slopes of the cost function at different levels but will usually not be able to cover the whole range of costs.
Cost function estimation

Idea here is to use observed cost data as a function of outputs to evaluate presence of cost synergies. Issue raised by definition 1 is that would ideally need to observe cost for individual products (i.e. observe single product firms). In practice this is rarely the case so that researchers typically rely on parametric assumption on cost function to estimate those costs. This is an issue and in practice different papers have reached different conclusions re the existence of economies of scope in the local and national telephony in the US following the break-up of AT&T in 1984 and the separation of local from long distance services.

Beresteanu (2005) proposes an alternative non parametric test that relies on a weaker condition that "economies of scope", "cost complementarity".

Definition 2: $C$ exhibits cost complementarities on $A$ is $C$ is submodular in $A$, i.e. if for any $Y \in A$, $Y' > 0$ such that $Y + Y'$, $Y + P_1 Y'$ and $Y + P_2 Y' \in A$,

$$C(Y + Y') - C(Y) \leq [C(Y + P_1 Y') - C(Y)] + [C(Y + P_2 Y') - C(Y)]$$
Advantage is that data requirement is smaller (you do not need to observe single product firms). Applies method to data on telephony and concludes that there exist cost complementarities between local and long distance telephony.
3 Multi-unit demand consumers

3.1 Demand

Empirical models of consumer level demand that we have seen so far, assume that each consumer consumes at most one unit of a single product in every observation period. In order to study firm behavior in the presence of multi-unit demand consumers, we now relax this assumption. As we do so, the main concern will be to understand the data requirement for estimating demand that flexibly incorporates substitution and complementarity patterns across goods.

3.1.1 Consumers may buy single units of several goods

Suppose there are two goods $A$ and $B$. Income not spent on $A$ or $B$ is spent to purchase a continuous compositive commodity. Utilities from each options are given
by:

\[ u_A = \delta_A - \alpha p_A + \nu_A \]

\[ u_B = \delta_B - \alpha p_B + \nu_B \]

\[ u_{AB} = u_A + u_B + \Gamma \]

\[ u_0 = 0 \text{ (normalization)} \]

where \([\nu_A, \nu_B]'\) are unobserved product qualities distributed according to a normal \(N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \sigma \\ \sigma & 1 \end{bmatrix}\right)\) and \(\Gamma\) is defined as \((u_{AB} - u_A) - (u_B - u_0)\).

**Definition** (Samuelson, 1974): Two goods are substitutes if the cross-price elasticity of compensated demand is positive. The two goods are complements otherwise.
In this simple setting, there is no wealth effect and the quality of substitutes or complements depends on the cross-elasticity of demand. Moreover it can be shown that cross-elasticities are positive (goods A and B are substitutes) iff $\Gamma < 0$, goods are complements if $\Gamma > 0$, and independent otherwise.

The model has 5 parameters: $\delta_A$, $\delta_B$, $\alpha$, $\Gamma$ and $\sigma$. The question is whether these parameters can be separately identified in the data (the concern is that observing consumers choose both $A$ and $B$, or none, could be due to either a high value for $\sigma$ or a high value of $\Gamma$). Without variations in the data, we only observe three independent moment conditions: $P_A$, $P_B$ and $P_{AB}$. So the model is not identified.

Identification can come from different sources of variation in the data: (1) variation in prices or explanatory variables entering $\delta_A$ or $\delta_B$ but not both (for example, a cross-section), (2) panel data and a parametric assumption according to which $\nu_{jt} = \nu_j + \varepsilon_{jt}$ where $\varepsilon_{jt}$ is i.i.d. across time and time. See Gentskow (2007) for a discussion and an application to online and offline news. In principle, demand for such a flexible model could rely on only aggregate data as long as we can observe the proportion of consumers buying both goods.
Suppose there is a single good but consumers may want to purchase several units of that good. The success or failure of any pricing scheme by the firm will depend on how individual demand varies with quantities. Without strong functional form assumptions, it should be clear that how individual demand varies with quantities cannot be identified from aggregate data.

Idea is to subdivide each agent (or firm) \( f \) as the aggregation of \( J_f \) independent decision makers making a decision for a specific task \( j \). The task involves the acquisition of a single unit of a product (or the outside good). The utility from the choice of option \( k \) for task \( j \) of firm \( f \) is given by

\[
U_{fjk} = \beta X_{kjf} + \delta_{kj} - \alpha p_k + \varepsilon_{fjk}
\]

The model allows for preferences for a computer brand at the firm level but also allows a firm to buy computers of different brands for the different tasks (observable task characteristics include divisions for example). However, the model rules out any other interactions among purchases for different tasks.
Let $x_{fjk} = 1$ if firm $f$ chooses product $k$ for task $j$. Suppose that the number of tasks within a firm is distributed according to $F_{J}(., | D_f, \theta)$ (which depends on parameters and firm characteristics $D_f$). The predicted number of computers bought by firm $j$ is equal to

$$\sum_{J_f=0}^{\infty} \left[ \sum_{j=0}^{J_f} \sum_{k} x_{fjk}(\theta, X) \right] \Pr(J_f | D_f, \theta)$$

Hendel (1999) applies some extension of this model to the demand for computers by the corporate sector. Dubé (2004) applies this idea to Carbonated soft drinks. The output of the empirical analysis is an individual demand for the products that allows the purchase of several units of the same products and the purchase of different products.
3.2 Pricing

When consumers buy several goods or several units of the same goods, firms can generally do better than linear prices. We say that a firm price discriminates when the prices it charges for consecutive units of the same good or across several of its products cannot be explained by marginal costs variations.

3.2.1 Bundling

Bundling refers to the practice of selling several products together for a different price than component products (in case of mixed bundling) or sell the products only as a bundle (pure bundling). Bundling is a form of (second degree) price discrimination that allows firms to extract more surplus from consumers. Adams and Yellen (1976) provided the basic intuition for why bundling may raise profits. Suppose a monopolist produces two goods, both a zero marginal costs. There are
two types of consumers. Type 1 have valuation 3 for good 1 and valuation 4 for good 2, type 2 have valuation 5 for good 1 and valuation 2.5 for good 2.

<table>
<thead>
<tr>
<th></th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_{12}$</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opt. linear prices</td>
<td>3</td>
<td>2.5</td>
<td>-</td>
<td>11</td>
</tr>
<tr>
<td>Opt. bundling</td>
<td>-</td>
<td>-</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

McAfee, McMillan, Whinston (1989) derived a sufficient condition for mixed bundling to be superior to linear prices (graphical intuition). More on this, next class.
3.2.2 Two-part tariffs and other nonlinear price schedules

Suppose consumers get utility $\theta V(q) - T$ when they consume $q$ units of the product and pay $T$ for it (they get zero otherwise). Assume that $V' > 0$ and $V'' < 0$. Suppose further that $\theta$ is privately observed by consumers and can take two values $\theta_L, \theta_H$ with $c < \theta_L < \theta_H$ (and that low types have proba $\lambda$).

At any linear price $p$, the demand from types $\theta$ is given by $\arg \max_q \theta V(q) - pq$. The FOC $\theta V'(q) = p$ condition implicitly defines a downward sloping demand function $D_{\theta}(p)$. Aggregate demand is given by $D(p) = \lambda D_{\theta_L}(p) + (1 - \lambda) D_{\theta_H}(p)$.

**Under linear pricing**, the firm chooses its price to maximize $(p - c)D(p)$.

**With a two-part tariff**, for any given $p$, the highest profit the firm achieve is when it sets the fixed component equal to $S_{\theta_L}(p) = \max_q \theta_L V(q) - pq$ (assuming that $S_{\theta_L}(p) > \lambda S_{\theta_H}(p)$). The optimal price in a two part tariff is then given as the price that maximizes $S_{\theta_L}(p) + (p - c)D(p)$.
Profits under a two-part tariff are higher, prices are lower (applications: telephone, gas, electricity, taxi, amusement park, membership yield discount on cultural events, ....).

Two-part tariffs can be generalized to multi-part tariffs or more generally non linear prices. It is easy to reinterpret quality in section 2.2. as quantity and thus view the model as one of optimal non-linear prices. By definition \( D(q, \theta) = u_q(q, \theta) \) and thus the FOC

\[
\begin{align*}
    u_q(q(\theta), \theta) &= C_q(q(\theta)) + \frac{1 - F(\theta)}{f(\theta)} u_{\theta q}(q(\theta), \theta)
\end{align*}
\]

can be rewritten as

\[
\begin{align*}
    D(q(\theta), \theta) - C_q(q(\theta)) &= \frac{1 - F(\theta)}{f(\theta)} D_\theta(q(\theta), \theta)
\end{align*}
\]

or

\[
\begin{align*}
    \frac{P'(q(\theta)) - C_q(q(\theta))}{P'(q(\theta))} &= \frac{1 - F(\theta)}{f(\theta)} D_\theta = \frac{1}{\eta(q(\theta), \theta)}
\end{align*}
\]
Wilson (1993) shows that the monopolist can very close to the profit from optimal non linear prices just with a 3 or 4 part tariffs.

3.2.3 Pricing in competitive environments

As before competition reduces distortion. Existing results are mainly derived in special cases.
3.3  Economies of scope

In the presence of bundling we usually richer price information (prices on bundles or prices for different quantitites that help us identify cost synergies. Consider the context of procurement auctions (multi-products, with buyer interested in several products) we can use the FOCs for optimal prices to infer marginal costs for component items:

\[
\max_{p_1, p_2, p_{12}} (p_1 - c_1) D_1(p_1, p_2, p_{12}) + (p_2 - c_2) D_2(p_1, p_2, p_{12}) + (p_{12} - c_{12}) D_{12}(p_1, p_2, p_{12})
\]

\[
\begin{bmatrix}
\frac{\partial D_1}{\partial p_1} & \frac{\partial D_1}{\partial p_2} & \frac{\partial D_{12}}{\partial p_1} \\
\frac{\partial D_1}{\partial D_1} & \frac{\partial D_1}{\partial D_2} & \frac{\partial D_{12}}{\partial D_1} \\
\frac{\partial D_{12}}{\partial D_1} & \frac{\partial D_{12}}{\partial D_2} & \frac{\partial D_{12}}{\partial D_{12}} \\
\end{bmatrix}
\begin{bmatrix}
p_1 - c_1 \\
p_2 - c_2 \\
p_{12} - c_{12}
\end{bmatrix}
= -
\begin{bmatrix}
D_1 \\
D_2 \\
D_{12}
\end{bmatrix}
\]

If Jacobian is invertible, \(c = p + \nabla D^{-1}D\)

Cantillon and Pesendorfer (2006) show that Jacobian is indeed invertible if \(D_j(p_1^*, p_2^*, p_{12}^*) > 0\) for all \(j\) (otherwise bounds).