

Empirical Likelihood

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CHAPMAN & HALL/CRC

Boca Raton London New York Washington, D.C.

Library of Congress Cataloging-in-Publication Data

Owen, Art. (Art B.)

Empirical likelihood / Art Owen.

p. cm.— (Monographs on statistics and applied probability ;
92)

Includes bibliographical references and index.

ISBN 1-58488-071-6 (alk. paper)

1. Estimation theory. 2. Probabilities. 3. Mathematical
statistics. I. Title. II. Series.

QA276.8 .094 2001

519.5'44—dc21

2001028680

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International Standard Book Number 1-58488-071-6

Library of Congress Card Number 2001028680

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

TO PATRIZIA, GREGORY, AND ELLIOT

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Preface

Empirical likelihood is a nonparametric method of inference based on a data-driven likelihood ratio function. Like the bootstrap and jackknife, empirical likelihood inference does not require us to specify a family of distributions for the data. Like parametric likelihood methods, empirical likelihood makes an automatic determination of the shape of confidence regions; it straightforwardly incorporates side information expressed through constraints or prior distributions; it extends to biased sampling and censored data, and it has very favorable asymptotic power properties. Empirical likelihood can be thought of as a bootstrap that does not resample, and as a likelihood without parametric assumptions.

This book describes and illustrates empirical likelihood inference, a subject that is ready for a book although it is still undergoing active development. Most of the published literature has emphasized mathematical study of asymptotics and simulations of coverage properties. This book emphasizes analyzing data in ways that illustrate the power and flexibility of empirical likelihood inference. The presentation is aimed primarily at students and at practitioners looking for new ways to handle their data. It is also aimed at researchers looking for new challenges.

The first four chapters form the core of the book. Chapters 5 through 8 extend the ideas to problems such as smoothing, biased sampling, censored and truncated data, confidence bands, time series, and finite populations. Chapter 9 relates empirical likelihood to other methods, and presents some hybrids. Chapter 10 describes some challenges and results near the research frontier. Chapters 11 through 13 contain proofs, computational details, and more advanced theory, respectively. An appendix collects some background material. A course in empirical likelihood could be designed around Chapters 1 through 4, supplemented with those other topics of most interest to the instructor and students.

Much more could have been written about some aspects of empirical likelihood. Applications and theory relevant to survival analysis and to econometrics come to mind, as do recent developments in kernel smoothing and finite population sampling.

The mathematical level of the text is geared towards students and practitioners, and where possible, the presentation stays close to the data. In particular, measure theoretic subtleties are ignored. Some very short and simple proofs are embedded in the text. Longer or more technical theoretical discussions are confined to their own chapters. Finally, the most difficult results are only outlined, and the reader

is referred to the literature for the details. A parallel triage has been applied to computational issues.

The worked examples in the text all use real data, instead of simulated, synthetic, or hypothetical data. I believe that all of the statistical problems illustrated are important ones, although in some examples the method illustrated is not one for which the data were gathered. The reader is asked to indulge some statistical license here, and to imagine his or her own data in the place of the illustrating data.

There is an empirical likelihood home page. At the time of writing the URL for that page is:

<http://www.stanford.edu/~owen/empirical>

The web site is for software, images, and other information related to empirical likelihood.

As an undergraduate, I studied statistics and computer science at the University of Waterloo. The statistics professors there instilled in me a habit of turning first to the likelihood function, whenever an inference problem appeared. I arrived at Stanford University for graduate study at a time when there was a lot of excitement about nonparametric methods. Empirical likelihood is a way of remaining in both traditions.

The idea for empirical likelihood arose when Rupert Miller assigned a problem in survival analysis from Kalbfleisch & Prentice (1980). The problem was to work out the nonparametric likelihood ratio inferences for the survival function as described in Thomas & Grunkemeier (1975). Around that time, there was a debate among some statistics students as to whether nonparametric confidence intervals for a univariate mean should point in the direction that the data seemed to be skewed, or in the opposite direction. There were intuitive arguments and existing methods to support either choice. I looked into empirical likelihood to see if it might point the way, and was surprised to find a nonparametric analog of Wilks's theorem, with the same distribution as in parametric settings. I now call these ELTs (empirical likelihood theorems) after a referee remarked that they are not Wilks's theorem.

Empirical likelihood has been developed by many researchers, as is evident from the bibliographic notes in this text. It is hard to identify only a few contributions from the many, and leave some others out. But it would be harder still not to list the following: Peter Hall, Tom DiCiccio, and Joe Romano obtained some very difficult and significant results on higher order asymptotics. These include Bartlett correctability, signed root corrections, pseudo-likelihood theory, bootstrap calibration, and connections to least favorable families. Jing Qin is responsible for many very creative problem formulations mixing empirical and parametric likelihood, combining multiple biased samples, and with Jerry Lawless, establishing results on using empirical likelihood with side information. Per Mykland has shown how to handle dependent data in a martingale setting. Empirical likelihood for censored and truncated data has been investigated by Gang Li, Susan Murphy,

and Aad van der Vaart. Yuichi Kitamura developed connections between empirical likelihood and modern econometrics, and has studied the large deviations properties of empirical likelihood.

I would like to thank Kirsty Stroud, Naomi Lynch, Hawk Denno, and Evelyn Meany of Chapman & Hall/CRC Press for watching over this book through the production process. It is a pleasure to acknowledge the National Science Foundation for supporting, in part, the writing of this book. I also thank the referees and editors who handled the early empirical likelihood papers. They were constructive and generous with their comments. I thank Dan Bloch, Richard Gill, Ker-Ai Lee, Gang Li, Hal Stern, and Thomas Yee, who sent me some data; Judi Davis for entering some data; Ingram Olkin for some tips on indexing; Tomas Rokicki, Eric Sampson, Rob Tibshirani, and C. L. Tondo for some pointers on LaTeX and related topics; Philip Gill, Michael Saunders, and Walter Murray for discussions over the years on nonlinear optimization; George Judge for conversations about econometrics; and Balasubramanian Narasimhan, who kept the computers humming and always seemed to know what software tool I should learn next. I owe a debt to Jiahua Chen, David Cox, Nancy Glenn, Fred Hickernell, David Hinkley, Kristopher Jennings, Li-Zhi Liao, Terry Therneau, and Thomas Yee for help in proofreading. Of course, I am responsible for any flaws that remain. Finally, and most of all, I thank my wife, Patrizia, and my sons, Gregory and Elliot, for their patience, understanding, and encouragement while this book was being written.

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May 2001