Graduate Microeconomics I
Problem Set 6

Fall 2009 - ECARES

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1. Consider a pure exchange economy with two individuals (A and B) and two goods (x and y). The utility functions are given by

$$U_A(x_A, y_A) = \min[x_A, y_A]$$
$$U_B(x_B, y_B) = \min[x_B, 2y_B],$$

where x_i and y_i are the quantities of the two goods consumed by individual $i = A, B$. The total endowments are $w_x = 10$ and $w_y = 5$.

   (a) Represent the indifference curves of both individuals in the Edgeworth box and find the Pareto set.

   (b) Let the individual endowments be $w_A = (7, 2)$ and $w_B = (3, 3)$. Determine the equilibrium quantities and prices?

   (c) What can we say about the equilibrium prices for other values of individual endowments?

2. Consider the following exchange economy: Antonio has a utility function $u_A(x_1, x_2) = x_1 + x_2$ and Bruno has a utility function $u_B(x_1, x_2) = \max\{x_1, x_2\}$. The total endowment for each of the goods is equal to 10.

   (a) Illustrate this situation in an Edgeworth box diagram and comment.

   (b) Determine the set of Pareto efficient allocations and represent them in a Edgeworth box diagram.

   (c) Assume that the endowment vectors are $w_A = (5, 5)$ and $w_B = (5, 5)$. Determine the competitive (Walrasian) equilibrium if exists.

(d) Assume that the endowment vectors are $w_A = (4, 1)$ and $w_B = (6, 9)$. Determine the competitive (Walrasian) equilibrium if exists.

3. There are two individuals who derive utility from food (x) and leisure (l). The utility functions are $U_1(x_1, l_1) = x_1^{1/2} l_1^{1/2}$ and $U_2(x_2, l_2) = x_2^{3/4} l_2^{1/4}$. Each individual is endowed with five hours of time, which they can use for leisure or labor supply (for which they receive a competitive wage). The food can be produced from labor by the single firm which uses the following technology: $x = 2\sqrt{L}$, where L is aggregate labor with $L = 10 - l_1 - l_2$. The firm is jointly owned by the individuals and the profits are shared equally between them.

   (a) Write the consumers' and producer's problems for this economy. Explain why we can normalize one of the prices.

   (b) Find the competitive equilibrium: quantities and prices.

   (c) Is this equilibrium is Pareto efficient?

   (d) Assume that the government controls the labor market and imposes a minimal nominal wage which is higher than the equilibrium wage. How is this going to affect the welfare of the individuals? Explain.

4. Suppose there are perfectly competitive markets with only two individuals, Philippe and Irina, and two goods, twinkies (T) and guitar playing time (G). Irina likes the way Philippe plays and enjoys it when Philippe plays the guitar. However, as with our usual assumptions under perfect competition, Irina is unable to set the price of guitars and takes Philippe's quantity of guitar playing as fixed. Philippe has a utility function $U_P(T_P, G_P) = T_P G_P$ and Irina has a utility function $U_I(T_I, G_I, G_P) = T_I G_I + 10G_P$. They are endowed with two twinkies each and two hours of guitar playing time each.

   (a) Find the competitive prices and allocation.

   (b) Is this a Pareto optimal allocation? [Hint: try having Irina give Philippe 1/10 of an hour of guitar playing time.]

   (c) If $U_I(T_I, G_I, G_P) = T_I G_I$, what are the Pareto optimal allocations?

5. Consider a pure exchange economy with two consumers, and two goods. The indirect utility functions of the agents are given by:

$$v^1(p_1, p_2, w^1) = \ln w^1 - a \ln p_1 - (1-a) \ln p_2$$
$$v^2(p_1, p_2, w^2) = \ln w^2 - b \ln p_1 - (1-b) \ln p_2,$$

with $0 < a < 1$ and $0 < b < 1$ and where p_i is the price of commodity $i = 1, 2$, while w_h is the monetary income of consumer $h = 1, 2$. The vectors of initial endowments are given by $w^1 = (2, 2)$ and $w^2 = (1, 1)$.

   (a) Calculate the market clearing prices.

   (b) Assume that the government organizes a transfer T from individual 1 to individual 2 (in other words, a lump sum tax T is imposed on individual 1 and a transfer T given to individual 2). Determine a condition on a and b under which this transfer does not a?ect the equilibrium prices. Comment.

   (c) Assume $a = b = 1/2$ and determine the level of T which equalizes equilibrium utility levels (in other words, T must yield $v^1 = v^2$, where the utility levels are evaluated at the market equilibrium induced by T).

6. Consider a simple Robinson-Crusoe economy. There is an initial endowment of one day of endowed time, T, per day of calender time. There is no leisure (i.e., leisure does not provide any utility to the consumer for some reason!). Time can be used to produce wine, x, or oysters, y. Let T_x denotes the time devoted to wine and T_y denote the time devoted to oysters. The production function for wine is $x = \sqrt{T_x}$, and that for oysters is $y = \sqrt{T_y}$. Preferences are represented by the utility function $U(x, y) = xy$.

   (a) Find the Pareto efficient allocation for this economy. Explain your method.

   (b) What are equilibrium prices that will support the efficient allocation as an equilibrium (assuming that the firm is owned by the unique consumer). You can set one of the prices arbitrarily at unity as numeraire.

   (c) Show that the prices derived in the previous point imply market clearing in all three markets in the economy.

7. Consider an economy with two goods: labor and a heating fuel. Good two (heating fuel) is produced either from a constant-returns technology (coal mine) given by $y_1 = z_1 \; y_1 \geq 0, \; z_1 \geq 0$, or from a decreasing returns technology (chopping down trees) given by $y_2 = 2\sqrt{z_2}, \; y_2 \geq 0, \; z_2 \geq 0$. y_j is the quantity produced by firm $j = 1, 2$; z_j is the quantity of labor used by firm j. The economy consists of two consumers whose consumption sets are given by :

$$X_1 = \{(l_i, x_i) | x_i \geq 0, 0 \leq l_i \leq 3\} \quad i = 1, 2$$

where l_i is the quantity of labor offered by consumer i, and x_i is the quantity of fuel consumed by consumer i. The preferences of consumer i are given by

$$U_i(l_i, x_i) = \frac{1}{2} \ln(3 - l_i) + \frac{1}{2} \ln x_i \quad i = 1, 2$$

(a) Characterize the feasible outcomes (allocations) in this economy.

(b) Define the Pareto optimal allocations for this economy (just state the problem at this stage; you will be asked to solve it below).

(c) Determine the aggregate production function of the economy. That is, find the best way to allocate a fixed amount of input $\bar{z}$ between the 2 technologies in order to maximize the total production, $y_1 + y_2$.

(d) Determine the set of Pareto optimal allocations. Show that all the Pareto optima are achieved from the same production plan.

(e) Now, suppose that the production sector is nationalized. The planner creates a conglomerate called "The Company of Fuels", which integrates the two firms. The conglomerate must choose its production plan to minimize its total cost for any given level of output (normalize the price of labor at one). It must price at average cost, and satisfy all the demand forthcoming at this price. Each consumer is free to sell his labor on the labor market and to buy fuel at its market price p. Characterize the resulting equilibrium. Is this solution optimal?

(f) Now the planner requires the conglomerate to price at marginal cost and to transfer a fraction $\alpha_i \in [0, 1]$, $(\alpha_1 + \alpha_2 = 1)$ of profits to consumer i. Characterize the equilibrium and show that when α_1 varies (from 0 to 1) the corresponding equilibria make up a strict subset of the set of Pareto optima. In what sense can we talk about competitive equilibrium?

Additionally, students are also encouraged to do the following exercises:

1. MWG 10.B.2
2. MWG 10.C.2
3. MWG 10.C.8
4. MWG 10.C.9
5. MWG 16.C.1
6. MWG 16.C.4
7. MWG 16.D.2
8. MWG 16.E.2
9. MWG 16.G.3
10. MWG 17.B.3
11. MWG 17.B.4
12. MWG 17.C.6
13. MWG 18.B.5